A SIMPLIFICATION FOR THE STUDY OF FISH POPULATIONS BY CAPTURE DATA

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ABSTRACT

Expressions given by Rafail for estimating catchability are modified here to eliminate iteration, for better accuracy, and a large economy in calculations and time. The evaluation of catchability allows the estimation of other important parameters with the useful assumption of their variabilities according to seasons and recognized sections of a population.

The evaluation of some parameters of fish populations from capture data began at the start of the century (Edser 1908; Heincke 1913; Baranov 1918). Beverton and Holt (1957) derived an equation in two forms (equations (14.19) and (14.86)) for the estimation of catchability and natural mortality from catch and effort data for a whole series of years assuming identical survival rates and catchabilities for all ages in a given year, fishing effort varies from year to year, and negligible recruitment and migrations.

Paloheimo (1961) modified the iteration method by Beverton and Holt (1957) to a simpler one without iteration using the relationship $(1 - e^{-i})/i \approx e^{-0.5i}$ where i is the instantaneous total mortality.

Allen (1966) described three methods for estimating a population and one for recruitment by using data on annual age composition, number caught, effort to take a known part of the catch assuming a constant recruitment rate all over a year, equal catchability for the different age groups, and available comparisons between exploited and unexploited populations with equal natural mortality. Allen (1968) described a simplification of his method for computing recruitment rates.

Among the investigators who studied the variability of parameters of fish populations, Gulland (1964) described variations in catchability as cyclical, long-term trends due to amount of fishing and changes in abundance, diurnal changes due to feeding and light, temperature like severe winters, and sex. Paloheimo and Kohler (1968) concluded from their analysis of a cod population that catchability and natural mortality showed variations associated with age and years. Walker (1970) gave evidence of increased natural mortality with age due to senescence for cod.

Rafail (1974) recognized the probable great variability of parameters of fish populations and derived expressions for the evaluation of catchability, fishing mortality, natural mortality, and recruitment assuming their variability from one season to another and their constancy during the seasons as well as their variation from a recognized section of a population to another like age-groups and different sexes. His equations for the evaluation of catchability as the first parameter to be estimated require a number of iterations which may be relatively very large if recruitments exceed the sum of natural and fishing mortalities. Therefore, a computer is needed for accurate calculations and this is a disadvantage.

The present treatment transforms the equations given by Rafail (1974) to estimate catchability into forms that dispense with iterations and yield more accurate estimates.

SAMPLING PROCEDURE

A fish population with a certain initial size is distributed on a constant area and subjected to a sequence of sampling surveys which can be grouped into a number of groups. Each group of surveys must contain at least three sampling surveys. The parameters of the population are assumed to vary among the groups of surveys and remain constant within each group which represents a season with constant properties. The entire fishing fleet may be considered as sampling

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vessels whose catch data are to be collected adequately.

If the fleet is large, a part of the fleet is appointed as sampling vessels while the effective fishing effort of unappointed vessels should be estimated. The sampling surveys should follow one another with no intervening time periods within a group of surveys. The durations of the surveys may vary from one survey to another or kept constant if the total fishing effort varys from survey to survey. The total effort exerted on the population should vary from one survey to another.

ASSUMPTIONS

1. A fish population has a constant area of distribution and a constant uniform distribution of fishing relative to fish concentration so that the instantaneous fishing mortality is proportional to fishing effort.

2. The population is subjected to a sequence of n sampling surveys grouped into M groups representing M seasons with constant population parameters. The duration of the kth survey is denoted by T_k. The catchability or percentage of available fish captured by a standard unit of fishing effort during the kth survey is denoted by q_k.

3. The commercial and sampling vessels exert a constant fishing effort per unit time during the kth survey denoted by f_{kc} and f_{ks} respectively, so that the efforts exerted become

$$f_{ks} \cdot T_k = f'_{ks} \quad (1.1)$$
$$f_{kc} \cdot T_k = f'_{kc} \quad (1.2)$$
$$f_{ks} + f_{kc} = f_k \quad (1.3)$$
$$f'_{ks} + f'_{kc} = f'_k \quad (1.4)$$

where f'_{ks}, f'_{kc}, and f'_k represent the total fishing effort exerted by the sampling, commercial vessels, and the whole fleet, respectively, during the kth survey.

4. The instantaneous fishing mortalities per unit time by the sampling, commercial, and total fleet in the kth survey are denoted by F_{ks}, F_{kc}, and F_k, respectively. The instantaneous fishing mortalities during the kth survey (F'_{ks}, F'_{kc}, and F'_k) are evaluated as

$$F_{ks} \cdot T_k = F'_{ks} = q_k \cdot f'_{ks} \quad (1.5)$$
$$F_{kc} \cdot T_k = F'_{kc} = q_k \cdot f'_{kc} \quad (1.6)$$
$$F_k \cdot T_k = F'_k = q_k \cdot f'_k. \quad (1.7)$$

5. The fish population has an initial size denoted by N_0. The number of fish present at the start of the kth survey is N_{k0} while the number of fish present at the end of the kth survey or the start of the $(k+1)$th survey is $N_{(k+1)0}$.

6. The instantaneous natural mortality rate per unit time during the kth survey is M_k. The instantaneous natural mortality during the kth survey is

$$M_k \cdot T_k = M'_k. \quad (1.8)$$

7. The instantaneous recruitment rate per unit time during the kth survey relative to the number of fish present is R_k. The number of fish present at the end of the kth survey or the start of the $(k+1)$th survey when recruitment is acting solely is

$$N_{(k+1)0} = N_{k0} \cdot \exp(R_k \cdot T_k)$$
$$= N_{k0} \cdot \exp(R'_k), \quad (1.9)$$

that is,

$$R_k \cdot T_k = R'_k, \quad (1.10)$$

where R'_k denotes the instantaneous recruitment rate during the kth survey.

8. The instantaneous rate of change of fish abundance per unit time during the kth survey relative to the number of fish present is "A_k" which is the "instantaneous abundance coefficient" so that

$$N_{(k+1)0} = N_{k0} \cdot \exp(A_k \cdot T_k)$$
$$= N_{k0} \cdot \exp(A'_k). \quad (1.11)$$

In other words, $A_k \cdot T_k = A'_k$ and A'_k denotes the instantaneous change of abundance during the kth survey.

According to previous assumptions we have

$$A'_k = R'_k - M'_k - F'_k = (R_k - M_k - F_k)T_k \quad (1.12)$$

and

$$N_{(k+1)0} = N_{k0} \cdot \exp(A'_k)$$
$$= N_{k0} \cdot \exp(R'_k - M'_k - F'_k). \quad (1.13)$$

9. If the sampling surveys $(k-1)$, k, and $(k+1)$ belong to the same season,

$$R_{k-1} = R_k = R_{k+1} = \bar{R}_k \quad (1.14)$$
$$M_{k-1} = M_k = M_{k+1} = \bar{M}_k \quad (1.15)$$

and

$$q_{k-1} = q_k = q_{k+1} = \bar{q}_k \quad (1.16)$$

where $\bar{R}_k$, $\bar{M}_k$, and $\bar{q}_k$ are constant parameters per unit time during the $(k-1)$th, kth, and $(k+1)$th sampling surveys which should belong to the same season.

$$\therefore \quad \bar{R}_k - \bar{M}_k = \bar{B}_k \text{ (a constant).} \quad (1.17)$$

10. If $T_k = T_{k-1} = T_{k+1}$ and similar to Equations (1.8), (1.10), and according to (1.17), we get

$$\bar{M}_k T_k = \bar{M}'_k,\ \bar{R}_k T_k = \bar{R}'_k, \text{ and } \bar{B}_k T_k = \bar{B}'_k \quad (1.18)$$

where $\bar{M}'_k$, $\bar{R}'_k$, and $\bar{B}'_k$ represent the instantaneous rates of natural mortality, recruitment, and the difference between them during single surveys (not per unit time) belonging to the same season when the durations of the surveys are made equal.

11. The number of fish captured by the sampling, commercial, and the total fleet during the kth survey are denoted by C_{ks}, C_{kc}, and C_k, respectively.

12. The catch per unit efforts during the kth survey obtained from sampling, commercial, and total fleet are respectively

$$(C/f')_{ks},\ (C/f')_{kc}, \text{ and } (C/f')_k$$

where f is primed (f') according to previous notations to designate exerted effort during a whole sampling survey and not per unit time.

13. The following expressions are used to obtain simpler mathematical equations:

$$\left(\exp(A'_k) - 1\right)/A'_k = a_k \quad (1.19)$$

$$a_k^2/a_{k-1} \cdot a_{k+1} = a'_k \quad (1.20)$$

$$(C/f')_k^2/(C/f')_{k-1} \cdot (C/f')_{k+1} = (C/f)'_k. \quad (1.21)$$

A MODIFICATION FOR THE EXPRESSION ESTIMATING CATCHABILITY

Rafail (1974) developed an estimate for $\bar{q}_k$ according to his equation (4.16) briefly as follows when the whole fleet is engaged for sampling:

$$C_k = N_0 \cdot \exp\left(\sum_{j=1}^{k-1} A'_j\right) \cdot F'_k \cdot a_k \quad (2.1)$$

and

$$C_{k+1} = N_0 \cdot \exp\left(\sum_{j=1}^{k} A'_j\right) \cdot F'_{k+1} \cdot a_{k+1} \quad (2.2)$$

$$\therefore \quad \frac{C_{k+1}}{C_k} = \exp(A'_k) \cdot \frac{a_{k+1}}{a_k} \cdot \frac{F'_{k+1}}{F'_k} \quad (2.3)$$

and

$$\frac{C_k}{C_{k-1}} = \exp(A'_{k-1}) \cdot \frac{a_k}{a_{k-1}} \cdot \frac{F'_k}{F'_{k-1}} \quad (2.4)$$

and

$$\frac{C_k^2}{C_{k-1} \cdot C_{k+1}} = \frac{\exp(A'_{k-1})}{\exp(A'_k)} \cdot \frac{a_k^2}{a_{k-1} \cdot a_{k+1}} \cdot \frac{F'^2_k}{F'_{k-1} \cdot F'_{k+1}}. \quad (2.5)$$

According to Equations (1.7) and (1.16) we get

$$\frac{F'^2_k}{F'_{k-1} \cdot F'_{k+1}} = \frac{\bar{q}_k^2 \cdot f'^2_k}{\bar{q}_k^2 \cdot f'_{k-1} \cdot f'_{k+1}} = \frac{f'^2_k}{f'_{k-1} \cdot f'_{k+1}}. \quad (2.6)$$

As we have $\frac{\exp(A'_{k-1})}{\exp(A'_k)} = \exp(A'_{k-1} - A'_k)$ and according to Equation (1.12), we get

$$\exp(A'_{k-1} - A'_k) = \exp\left((R_{k-1} - M_{k-1} - F_{k-1})T_{k-1} - (R_k - M_k - F_k)T_k\right).$$

Again according to Equations (1.14) and (1.15), as well as (1.7) and (1.16), we get

$$\exp(A'_{k-1} - A'_k) = \exp\left((\bar{R}_k - \bar{M}_k)(T_{k-1} - T_k) - \bar{q}_k(f'_{k-1} - f'_k)\right). \quad (2.7)$$

From Equations (1.20), (2.5), (2.6), and (2.7) we get

$$\frac{C_k^2}{C_{k-1} \cdot C_{k+1}} = \exp\left((\bar{R}_k - \bar{M}_k)(T_{k-1} - T_k) - \bar{q}_k(f'_{k-1} - f'_k)\right) \cdot a'_k \cdot \frac{f'^2_k}{f'_{k-1} \cdot f'_{k+1}}.$$

Rearranging and according to assumption 12 we get

$$\frac{(C/f')_k^2}{(C/f')_{k-1} \cdot (C/f')_{k+1}} = \exp\left((\bar{R}_k - \bar{M}_k)(T_{k-1} - T_k) - \bar{q}_k(f'_{k-1} - f'_k)\right) \cdot a_k.$$

Using Equation (1.21), the above equation is transformed to

$$\bar{q}_k = \frac{\log_e(a_k') + (\bar{R}_k - \bar{M}_k)(T_{k-1} - T_k) - \log_e(C/f)_k'}{f_{k-1}' - f_k'}. \tag{2.8}$$

If sampling surveys are arranged to have equal durations (or $T_{k-1} = T_k = T_{k+1}$), then Equation (2.8) reduces to

$$\bar{q}_k = \frac{\log_e(a_k') - \log_e(C/f)_k'}{f_{k-1}' - f_k'}. \tag{2.9}$$

Equations (2.8) and (2.9) will be modified if a part of the commercial fleet is engaged with the sampling surveys so that $(C/f)_k'$ will be replaced by $(C/f)_{ks}'$, so that the last expression will be evaluated from the catch per unit effort of the sampling vessels "$(C/f')_{ks}$ of assumption 12," while all other items will remain the same.

Again it is important to note that the data of three successive surveys should be used to obtain a single q-estimate because in case of unsuccessive data the fraction $\exp(A_{k-1}')/\exp(A_k')$ of Equation (2.5) will be biased and Equations (2.8) and (2.9) will not hold good.

Equations (2.8) and (2.9) can be used to estimate $\bar{q}_k$ by a number of iterations which is large when fish abundance is increasing and much fewer with decreasing abundance (Rafail 1974).

The modification of Equations (2.8) and (2.9) is based on the fact that a_k shown by Equation (1.19) can be evaluated as a function of A_k'. Paloheimo (1961) gave the following approximation:

$$a_k = \left(1 - \exp(-A')\right)/A' \approx \exp(-0.5A'). \tag{3.1}$$

Rafail (1974) has shown that when the instantaneous rate of change of fish abundance is negative, then a_k of Equation (1.19) can be represented as in Equation (3.1). In fact a_k is more precisely expressed as

$$a_k \approx \exp(\alpha_1 A_k' + \alpha_2 A_k'^2 + \alpha_3 A_k'^3) \tag{3.2}$$

where α_1, α_2, and α_3 denote certain constants. A simpler and sufficient precise expression for a_k is fitted here as

$$a_k \approx \exp(\pm 0.5A_k' + 0.04A_k'^2). \tag{3.3}$$

Table 1 shows a comparison between the values of a_k calculated by the exact Equation (1.19) and those calculated by Equation (3.3).

TABLE 1.—A comparison between a_k-values calculated according to the exact Equations (1.19) and (3.3).

A'	$\exp(A')$	$a = (\exp(A') - 1)/A'$	$x = \pm 0.5A' + 0.04A'^2$	$a = \exp(x)$
−0.02	0.9802	0.9901	−0.01	0.9900
−0.10	0.9048	0.9516	−0.0496	0.9516
−0.20	0.8187	0.9063	−0.0984	0.9063
−0.50	0.6065	0.7869	−0.2400	0.7866
−1.00	0.3679	0.6321	−0.4600	0.6313
−2.00	0.1353	0.4323	−0.8400	0.4317
−2.25	0.1054	0.3976	−0.9225	0.3975
−2.50	0.0821	0.3672	−1.00	0.3679
−2.65	0.0707	0.3507	−1.0441	0.3520
−2.75	0.0639	0.3404	−1.0725	0.3421
−3.00	0.0498	0.3167	−1.14	0.3198
0.02	1.0202	1.0100	0.010016	1.0107
0.10	1.1053	1.0530	0.05040	1.0517
0.20	1.2215	1.1075	0.10160	1.1070
0.50	1.6486	1.2972	0.26000	1.2968
1.00	2.7184	1.7184	0.54000	1.7160
2.00	7.3890	3.1945	1.16000	3.1900
2.25	9.4877	3.7723	1.32750	3.7716
2.50	12.1828	4.4731	1.50000	4.4817
2.65	14.1544	4.9639	1.60590	4.9823
2.75	15.6428	5.3246	1.67750	5.3521
3.00	20.087	6.3623	1.86000	6.4237

Table 1 shows that Equation (3.3) can be used to calculate a_k with a maximum error less than 1% when A' lies between ± 3.00, i.e., an error which is practically negligible. Again, the smaller the value of A' the smaller is the error so that when A' lies between ± 2.5, the error is less than 0.2%, and Equation (3.3) can be considered as a highly precise expression in that range which is always encountered in fisheries studies. Equation (3.3) can be used to evaluate a_k' given by Equation (1.20) as

$$a_k' = \frac{\left(\exp(\alpha_1 A_k' + \alpha_2 A_k'^2)\right)^2}{\exp(\alpha_1 A_{k-1}' + \alpha_2 A_{k-1}'^2) \cdot \exp(\alpha_1 A_{k+1}' + \alpha_2 A_{k+1}'^2)}$$

and

$$\log_e a_k' = \alpha_1(2A_k' - A_{k-1}' - A_{k+1}') + \alpha_2(2A_k'^2 - A_{k-1}'^2 - A_{k+1}'^2). \tag{4.1}$$

According to Equations (1.12), (1.14), (1.15), and (1.16) we get

$$A_k' = (\bar{R}_k - \bar{M}_k)T_k - F_k' \tag{4.2}$$

$$\begin{aligned} \therefore 2A_k' - A_{k-1}' - A_{k+1}' &= 2T_k(\bar{R}_k - \bar{M}_k) - 2F_k' \\ &\quad - T_{k-1}(\bar{R}_k - \bar{M}_k) + F_{k-1}' \\ &\quad - T_{k+1}(\bar{R}_k - \bar{M}_k) + F_{k+1}' \\ &= (\bar{R}_k - \bar{M}_k)(2T_k - T_{k-1} - T_{k+1}) \\ &\quad - 2F_k' + F_{k-1}' + F_{k+1}' \end{aligned}$$

or

$$2A'_k - A'_{k-1} - A'_{k+1} = (\bar{R}_k - \bar{M}_k)(2T_k - T_{k-1} - T_{k+1}) - \bar{q}_k(2f'_k - f'_{k-1} - f'_{k+1}). \quad (4.3)$$

Denoting

$$\alpha_2(2A'^2_k - A'^2_{k-1} - A'^2_{k+1}) \quad (4.4)$$

of Equation (4.1) by $\phi A'$.

Equations (4.3) and (4.4) can be used to evaluate $\log_e a'_k$ given by Equation (4.1) as

$$\log_e a'_k = \alpha_1(\bar{R}_k - \bar{M}_k)(2T_k - T_{k-1} - T_{k+1}) - \alpha_1\bar{q}_k(2f'_k - f'_{k-1} - f'_{k+1}) + \phi A'. \quad (4.5)$$

Equation (4.5) can be inserted in Equation (2.8) to have another expression for $\bar{q}_k$ as follows:

$$\bar{q}_k(f'_{k-1} - f'_k) = \phi A' + \alpha_1(\bar{R}_k - \bar{M}_k)(2T_k - T_{k-1} - T_{k+1}) - \alpha_1\bar{q}_k(2f'_k - f'_{k-1} - f'_{k+1}) + (\bar{R}_k - \bar{M}_k)(T_{k-1} - T_k) - \log_e(C/f)'_k$$

or

$$\bar{q}_k(f'_{k-1} - f'_k + 2\alpha_1 f'_k - \alpha_1 f'_{k-1} - \alpha_1 f'_{k+1}) = \phi A' + [\bar{R}_k - \bar{M}_k][T_k(2\alpha_1 - 1) + T_{k-1}(1 - \alpha_1) - \alpha_1 T_{k+1}] - \log_e(C/f)'_k$$

or

$$\bar{q}_k = \frac{\phi A' + [\bar{R}_k - \bar{M}_k][T_k(2\alpha_1 - 1) + T_{k-1}(1 - \alpha_1) - \alpha_1 T_{k+1}] - \log_e(C/f)'_k}{f'_k(2\alpha_1 - 1) + f'_{k-1}(1 - \alpha_1) - \alpha_1 f'_{k+1}}. \quad (5.1)$$

According to Equation (3.3) we find that 0.5 is a very good estimate for α_1 which can be inserted in Equation (5.1) to obtain

$$\bar{q}_k = \frac{\phi A' + 0.5(\bar{R}_k - \bar{M}_k)(T_{k-1} - T_{k+1}) - \log_e(C/f)'_k}{0.5(f'_{k-1} - f'_{k+1})}. \quad (5.2)$$

If sampling surveys are carried out during equal time intervals, i.e., $T_{k-1} = T_k = T_{k+1}$; Equation (5.2) becomes

$$\bar{q}_k = \frac{\phi A' - \log_e(C/f)'_k}{0.5(f'_{k-1} - f'_{k+1})}. \quad (5.3)$$

Equation (3.3) shows that α_2 is estimated at 0.04 so that $\phi A'$ becomes according to Equation (4.4) as

$$\phi A' = 0.04(2A'^2_k - A'^2_{k-1} - A'^2_{k+1}). \quad (5.4)$$

The correction term $\phi A'$ given in Equation (5.4) can be put in another form by the inspection of the term A' shown by Equation (4.2)

$$A'_k = (\bar{R}_k - \bar{M}_k)T_k - F'_k.$$

The parameters $\bar{R}_k$ and $\bar{M}_k$ are supposed to be constant during any group of sampling surveys according to assumption 9, and Equation (1.17) we have

$$\bar{R}_k - \bar{M}_k = \bar{B}_k \quad \text{a constant}$$

$$\therefore \quad A'_k = \bar{B}_k T_k - F'_k \quad (5.5)$$

and

$$A'^2_k = \bar{B}_k^2 T_k^2 - 2\bar{B}_k T_k F'_k + F'^2_k \quad (5.6)$$

and $\phi A'$ of Equation (5.4) becomes

$$\phi A' = 0.04(\bar{R}_k - \bar{M}_k)^2(2T_k^2 - T^2_{k-1} - T^2_{k+1}) - 0.08(\bar{R}_k - \bar{M}_k)(2F'_k T_k - F'_{k-1}T_{k-1} - F'_{k+1}T_{k+1}) + 0.04(2F'^2_k - F'^2_{k-1} - F'^2_{k+1}). \quad (5.7)$$

If $T_k = T_{k-1} = T_{k+1}$ and according to Equation (1.18) we have

$$\bar{M}_k T_k = \bar{M}'_k \text{ and } \bar{R}_k T_k = \bar{R}'_k$$

$$\therefore \phi A' = -0.08(\bar{R}'_k - \bar{M}'_k)(2F'_k - F'_{k+1} - F'_{k+1}) + 0.04(2F'^2_k - F'^2_{k-1} - F'^2_{k+1}). \quad (5.8)$$

If Equation (3.2) is used to evaluate a_k,

$$a'_k \approx \frac{(\exp(\alpha_1 A'_k + \alpha_2 A'^2_k + \alpha_3 A'^3_k))^2}{\exp(\alpha_1 A'_{k-1} + \alpha_2 A'^2_{k-1} + \alpha_3 A'^3_{k-1}) \cdot \exp(\alpha_1 A'_{k+1} + \alpha_2 A'^2_{k+1} + \alpha_3 A'^3_{k+1})}$$

and

$$\log_e a_k' \approx \alpha_1(2A_k' - A_{k-1}' - A_{k+1}') + \alpha_2(2A_k'^2 - A_{k-1}'^2 - A_{k+1}'^2) + \alpha_3(2A_k'^3 - A_{k-1}'^3 - A_{k+1}'^3).$$

Following Equations (4.1) to (5.1) steps, we get an expression for $\bar{q}_k$ similar to Equation (5.1) with $\phi A'$ as

$$\phi A' = \alpha_2(2A_k'^2 - A_{k-1}'^2 - A_{k+1}'^2) + \alpha_3(2A_k'^3 - A_{k-1}'^3 - A_{k+1}'^3). \quad (5.9)$$

ESTIMATION OF CATCHABILITY

Denoting all terms of the numerators of Equations (5.1), (5.2), and (5.3) with the exception of $\log_e(C/f)_k'$ by "p" and their denominator by ϕF; the equations become

$$\bar{q}_k = \frac{-\log_e(C/f)_k' + p}{\phi f}. \quad (6.1)$$

Equating p to zero, a first estimate for $\bar{q}_k$ is obtained which is used together with catch data to estimate A', $\bar{R}_k$, $\bar{M}_k$, and $\phi A'$ so that p can be estimated and used to obtain the required estimate for $\bar{q}_k$ as well as other parameters.

If p has a negative sign, this means that the first estimate for $\bar{q}_k$ was higher than the true value and $p/\phi f$ is the correction to be subtracted to obtain the improved estimate and the reverse holds good as will be shown by the solved example. Equation (6.1) is therefore betterly transformed to

$$\bar{q}_k = \frac{-\log_e(C/f)_k'}{\phi f} + \frac{p}{|\phi f|}. \quad (6.2)$$

Solved examples showed that one single correction is sufficient to obtain precise estimates for $\bar{q}_k$ for populations with increasing or decreasing abundance which is a great advantage.

If a number of equations like (6.2) are available, they may be combined in a single expression as

$$\bar{q}_k = -\frac{\Sigma \log_e(C/f)_k'}{\Sigma\phi f} + \frac{\Sigma p}{\Sigma|\phi f|}. \quad (6.3)$$

EXAMPLE

Detailed informations are required to use the equations given above for estimating correctly the catchability as dividing sampling surveys into groups coinciding with seasons having more or less constant population parameters like periods with high, low, or nil recruitment, migration, natural mortality, and catchability.

As published data reviewed by the author lacked such information, it was decided to treat the hypothetical example given by Rafail (1974) so as to demonstrate the advantage of the above modified equations. Table 2 shows a part of 1974 example containing periods I and III with increasing and decreasing abundance, respectively.

Computations for Period I

A) Surveys 1, 2, and 3

$$\log_e(C/f)_k' = \log_e(1.00118) = 0.00116$$
$$\phi f = 0.5(1{,}000\text{–}2{,}000) = -500$$
$$\bar{q}_k = -0.00116/-500 = 2.320 \times 10^{-6}.$$

Above $\bar{q}_k$-estimate is used to evaluate A', $(\bar{R}_k' - \bar{M}_k)$, and $\phi A'$ using the relations:

$$F_k' = \bar{q}_k f_k',\ N_{k0} = \text{catch}/F_k'$$
$$\exp(A_k') = N_{k+1}/N_k$$
$$\bar{R}_k' - \bar{M}_k' = A_k' + F_k''$$
$$\hat{A}' = \bar{R}_k' - \bar{M}_k'^{xx} - F_k'.$$

TABLE 2.—A hypothetical example showing sampling periods I and III with increasing and decreasing abundance.

Period and survey k	Initial abundance N_{k_0}	Effort f_k	Total mortality	Abundance coefficient A_k'	$\exp(A_k')$	a_k	Catch $N_{k_0}F_k'a_k$
Period I		$\bar{q}_k = 2 \times 10^{-6}$	$\bar{M}_k' = 0.001$	$\bar{R}_k' = 0.450$			
1	1,000,000	1,000	0.003	0.447	1.5636	1.26085	2,522
2	1,563,600	3,000	0.007	0.443	1.5575	1.25847	11,807
3	2,435,307	2,000	0.005	0.445	1.5605	1.25955	12,269
4	3,800,297	4,000	0.009	0.441	1.5543	1.25692	38,212
Period III		$\bar{q}_k = 2 \times 10^{-6}$	$\bar{M}_k' = 0.020$	$\bar{R}_k' = 0.002$			
1	5,894,992	40,000	0.100	−0.098	0.90666	0.95245	449,175
2	5,344,753	20,000	0.060	−0.058	0.94365	0.97155	207,708
3	5,043,576	10,000	0.040	−0.038	0.96271	0.98132	98,985

where $\bar{R}'_k - \bar{M}'^{xx}_k$ is the mean of available values.

All the above relations are correct except the relation N_{k0} = catch/F'_k which is an approximation of N_{k0} = catch/$F'_k \cdot a_k$. If the computations show that the calculated $(\bar{R} - \bar{M})$-values are close to each other, then the approximate expression for N_{k0} is satisfactory to obtain accurate estimates for $\bar{q}_k$. Significantly different $(R - M)$-values may also lead to accurate estimates for $\bar{q}_k$. However, it may be necessary to use $\hat{A}'_k$ to estimate a_k to obtain improved estimates for N_{k0}-values to arrive at a better estimate for A'_k and $(\bar{R} - \bar{M})$-values. The rest of the computations for period I are:

K	F'_k	$N_{k0} = C/F'$	$\exp(A'_k)$	A'_k	$\bar{R}'_k - \bar{M}'_k$	$\hat{A}'_k$
1	2.32×10^{-3}	1,087,070	1.56503	0.44789	0.45021	0.44818
2	6.96×10^{-3}	1,696,408	1.55869	0.44378	0.45072	0.44354
3	4.64×10^{-3}	2,644,181			0.4505^{xx}	0.44586

According to Equation (5.4) we get

$$\hat{A}'^2_2 = 0.1967277, \hat{A}'^2_1 = 0.2008653,$$
$$\hat{A}'^2_3 = 0.1987911$$
$$\phi A' = 0.04(0.393455-0.399656)$$
$$= 0.04(-0.0062) = -0.000248$$
$$\phi A'/|\phi f| = -0.000248/500 = -0.496 \times 10^{-6}$$
$$\therefore \quad \bar{q}_k = (2.320-0.496)10^{-6} = 1.824 \times 10^{-6}.$$

According to Equation (5.8) we can calculate $\phi A'$ by another way as

$$\phi A' = -0.08(0.4505)(13.92-2.32-4.64)(10^{-3}) +0.04(96.88-5.38-21.53)(10^{-6})$$
$$= (-0.2508+0.0028)(10^{-3}) = -0.248(10^{-3}).$$

That is, the two methods gave the same results.

B) Surveys 2, 3, and 4

$$\log_e(C/f)'_k = 0.0009$$
$$\phi f = -500$$
$$\bar{q}_k = -0.0009/-500 = 1.8 \times 10^{-6}.$$

The following estimates are obtained by above steps

$$\bar{R}'_k - \bar{M}'^{xx}_k = 0.44782$$
$$\hat{A}'_2 = 0.44242, \hat{A}'_3 = 0.44422,$$
$$\hat{A}'_4 = 0.44062$$
$$\phi A' = 0.04(0.3946628-0.3898815)$$
$$= 0.000191$$

$$\therefore \phi A'/|\phi f| = 0.000191/500 = 0.382 \times 10^{-6}$$
$$\therefore \quad \bar{q}_k = (1.8+0.382)10^{-6}$$
$$= 2.182 \times 10^{-6}.$$

The arithmetic mean for $\bar{q}_k$ from the four surveys is

$$(1.824+2.182)(10^{-6})/2 = 4.006 \times 10^{-6}/2$$
$$= 2.003 \times 10^{-6}.$$

Equation (6.3) can be used to estimate $\bar{q}_k$ in one step as

$$\bar{q}_k = \frac{-0.00116+0.00090}{-1,000} + \frac{-0.000248+0.000191}{1,000}$$
$$= \frac{0.002060}{1,000} - \frac{0.000057}{1,000} = \frac{0.002003}{1,000}$$
$$= 2.003 \times 10^{-6}.$$

Period I has four sampling surveys and only two estimates for q can be obtained as the data of only three successive surveys are used to get a single q-estimate as explained above.

Computations for Period III

$$\log_e(C/f)'_k = -0.03012$$
$$\phi f = ½(40,000-10,000) = 15,000$$
$$q_k = \frac{-0.03012}{15,000} = 2.008 \times 10^{-6}.$$

The following computations are obtained according to the last estimate of catchability

K	$N_{k0} = C/F_k$	$(\bar{R}'_k - \bar{M}'_k)_1$	$\hat{A}'_1$	a_k	$N'_{k0} = C/F'_k a_k$
1	5,592,318	0.00214	−0.08322	0.9595	5,828,366
2	5,172,012	−0.00788	−0.04306	0.9786	5,285,113
3	4,929,531	−0.00290^{xx}	−0.02298	0.9887	4,985,871

$(\bar{R}'_k - \bar{M}'_k)_2$	$\hat{A}'_2$
−0.01763	−0.09819
−0.01811	−0.05803
−0.01787^{xx}	−0.03795

Above estimates show a recognizable variability for the first estimated $(\bar{R}'_k - \bar{M}'_k)_1$ parameters; so the calculations are proceeded to obtain the next $(\bar{R}'_k - \bar{M}'_k)_2$-estimates which are in fact highly accurate if compared with the original values in Table 2.

Using the so-called the less accurate $\hat{A}'_1$-estimates to calculate $\phi A'$; we get

$$\begin{aligned}\phi A' &= 0.04(0.0037080-0.0074535)\\ &= -0.00015\\ \phi A'/|\phi f| &= -0.00015/15{,}000 = -0.00001/1{,}000\\ &= -0.01 \times 10^{-6}\\ \bar{q}_k &= (2.008-0.010)10^{-6} = 1.998 \times 10^{-6}.\end{aligned}$$

Using the more accurate $\hat{A}_2'$-estimates we get

$$\begin{aligned}\phi A' &= 0.04(0.00673496-0.01108147)\\ &= -0.000174\\ \phi A'/|\phi f| &= -0.000174/15{,}000 = -0.011 \times 10^{-6}\\ \therefore \quad \bar{q}_k &= (2.008-0.011)10^{-6} = 1.997 \times 10^{-6}.\end{aligned}$$

Using Equation (5.8) and the more accurate $(\bar{R}_k' - \bar{M}_k')$-estimates, we get a similar result as

$$\begin{aligned}\phi A' &= -0.08(-0.01787)(-0.02008)\\ &\quad + 0.04(-0.0036289)\\ &= -0.0000287-0.0001451 = -0.000174.\end{aligned}$$

The above example shows that the so-called less accurate estimates gave equivalent results to the more accurate estimates. However, in situations with variable $(\bar{R}_k' - \bar{M}_k')$-values it will be preferable to compare their results with those to be obtained with the more accurate values.

DISCUSSION

Rafail (1974) showed the great advantages of his method for the estimation of some important parameters of fish populations like catchability, fishing mortality, natural mortality, and recruitment from catch data. He also showed that a similar analysis of data of tagged fish can allow the estimation of other important parameters like migrations and at the same time may correct the estimates of parameters of untagged fish that may be biased by unexpected recruitments and migrations.

The modifications presented here for expressions used to estimate catchability cause a great simplification, shortening of calculations and more accurate results. Rafail (1974) gave in his table 4 a summary of results of HP-20 computer programme for iteration of period I with increasing abundance. The results of the computer showed that after 16 iterations with a precision at six decimals and 22 iterations with a precision at nine decimals; q was estimated at 1.92×10^{-6} and 1.83×10^{-6}, respectively. The corresponding estimate by the present modified expressions was 1.824×10^{-6} by a single step. This simplification allowed the estimation of q from the next series of sampling surveys of period I (2, 3, and 4) so that an overall estimate of 2.003×10^{-6} becomes available which is highly accurate as the original value is 2×10^{-6}.

As far as period III with decreasing abundance is concerned, we find that 1974-expressions gave after three iterations 1.98×10^{-6} while the new expressions gave after one step 1.998×10^{-6} or 1.997×10^{-6} for q compared with an original value of 2×10^{-6}.

It is, therefore, concluded that the present modified expressions allow better accuracy and large economy in calculations and time during estimating q as compared with 1974-expressions. This greater accuracy of q will allow better estimates for other parameters. It appears what is a logic conclusion that the larger number of surveys, the larger will be the number of available q-estimates allowing a more accurate evaluation for catchability and other parameters.

SUMMARY

Modifications are presented here for expressions given by Rafail (1974) for estimating catchability to evaluate fishing and natural mortalities, recruitment, and migration assuming seasonal and subpopulation variability and the constancy of the parameters within the seasons. These modifications depend on the relation

$$(\exp(A_k') - 1)/A_k' = \exp(\pm 0.5A_k' + 0.04A_k'^2)$$

where A_k' denotes the instantaneous rate of change of fish abundance during the kth sampling period. The above expression is an extension of Paloheimo (1961) expression and gave a maximum error less than 1% when A' lies between ± 3.0 and smaller errors at smaller values of A so that the errors are less than 0.2% when A' lies between ± 2.5. This expression can be considered as highly accurate in the range that is always encountered in fisheries studies.

The modified expressions allow a large economy in calculations and time and a better accuracy for the estimation of catchability.

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