Optimal course by dolphins for detection avoidance

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One of the assumptions of line transect sampling is that movement of animals being counted is not in response to the approaching vessel before the animals are detected (Burnham et al. 1980). By observing from a helicopter the reaction of dolphins to an approaching survey vessel, Au and Perryman (1982) and Hewitt (1985) demonstrated that dolphin schools can detect the approach and maneuver to attempt to avoid detection. Because it may be that dolphin exhibit forms of optimal behavior (Au and Weihs 1980), it is of interest to determine whether there is a direction the dolphin should take that maximizes their distance to the vessel at the point of closest approach and, if there is such a direction, to determine whether dolphin use it. If this is so, this may be the way of determining through aerial means when dolphin first react to an approaching vessel and whether it is after they are detected by a shipboard observer.

Since the advent of purse-seine fishing in the eastern tropical Pacific in 1959, dolphin that associate with yellowfin tuna (i.e., primarily *Stenella attenuata*, *S. longirostris*, and *Delphinus delphis*) are chased, caught in nets, and sometimes drowned (Perrin 1968, 1969). Stuntz and Perrin (1979) reported that these species of dolphin are more difficult to capture in areas where purse-seine-vessel fishing effort has been greatest, implying that evasive behavior may be learned. It has also been reported by Au and Perryman (1982) that evasive maneuvers by dolphin upon approach of a vessel sometimes begin at a distance that is approximately the shipboard observer's horizon. Because the visual horizon of even a leaping dolphin is shorter than that of a shipboard observer, it is likely that they are reacting to the vessel sound. It is therefore plausible that by experiencing repeatedly the approach of such vessels, dolphin not only have learned to evade but do so optimally by choosing through trial and error the direction of escape, if it exists, in which the noise amplitude increases the least. Because the attenuation of sound is proportional to the distance transversed by it, escaping from a sound source in the direction where the amplitude increases the least is the same direction that maximizes the distance between a uniformly-moving source and receiver at the point of closest approach.

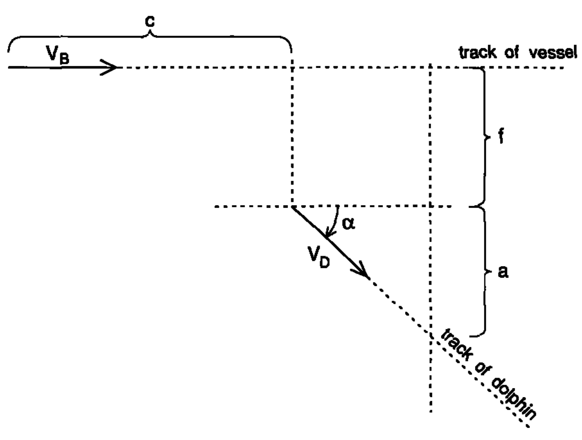

Figure 1
The vessel traveling at speed V_B and the dolphin traveling at V_D as seen in the stationary frame of reference. The direction α is the one chosen by the dolphin. Distance f is the perpendicular distance between the initial position of the dolphin and the projected path of the vessel, while a+f is the distance between vessel and dolphin when the dolphin is abeam.

Here, we formulate the following problem: Upon detecting the approach of a vessel, a dolphin attempts to avoid detection by retreating. If the velocity of the vessel is $\mathbf{V}_B$ and that of the dolphin is $\mathbf{V}_D$, is there a direction in which a dolphin can escape to maximize its distance from the vessel at the point of closest approach (Fig. 1)? And if so, what direction is it? We will show that there is such a direction: If α is the angle between $\mathbf{V}_B$ and $\mathbf{V}_D$, the angle $\alpha = \arccos(V_D/V_B)$, where V_B and V_D are, respectively, the speeds of the vessel and the dolphin, will maximize the distance

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between vessel and dolphin at the point of closest approach.

Finally, to ease the task of data analysis by whoever makes the necessary observations, we have derived the expressions that relate dolphin speed and direction to their range and bearing from the vessel. In a cartesian coordinate system let V_x and V_y be, respectively, the x and y components of the dolphin velocity minus, respectively, the x and y components of the vessel velocity. Both V_x and V_y are constructed using range and bearing measurements of the dolphin from the vessel. Then $V_D=[(V_x+V_B)^2+V_y^2]^{1/2}$ and $\alpha=\arctan[V_y/(V_x+V_B)]$.

Problem solution

As stated in the formulation above, the problem makes sense only for the case $V_D \leqslant V_B$. With reference to Figure 2, to maximize the distance between vessel and dolphin at the point of closest approach, we must find the maximum value of r_{min} with respect to angle $0 \leqslant \alpha \leqslant \pi$.

For the purpose of the following exposition, we define initial position to be that position of the vessel (or the dolphin) at the time when the dolphin detects the approaching vessel and begins evasion. Initial time is the time corresponding to the initial position.

In Figure 2, at the point of closest approach the distance between vessel and dolphin is given by

$$r_{min} = (a+f)\cos\beta, \quad (1)$$

where $0 \leqslant \beta \leqslant \pi/2$, f is the perpendicular distance between the initial position of the dolphin and the projected path of the vessel, and (a+f) is the distance between vessel and dolphin when the dolphin is abeam. The vector diagram of Figure 2 shows that the apparent track of the dolphin as seen from the vessel is a function of α, the direction of escape of the dolphin. Therefore, to solve the problem as posed, we must find the extreme value of Eq. (1) with respect to angle α.

By computing the derivative with respect to α of Eq. (1), we find that r_{min} is rendered an extreme value when

$$\frac{dr_{min}}{d\alpha} = \cos\beta\frac{da}{d\alpha} - (a+f)\sin\beta\frac{d\beta}{d\alpha} \quad (2)$$

vanishes. Depending on the functional dependence of a and β on α, an equation of the form of Eq. (2), could vanish either term by term or by cancellation of the terms. For the former case, each term could vanish trivially (i.e., a and β are independent of α), or nontrivially (i.e., both a and β are rendered extreme values

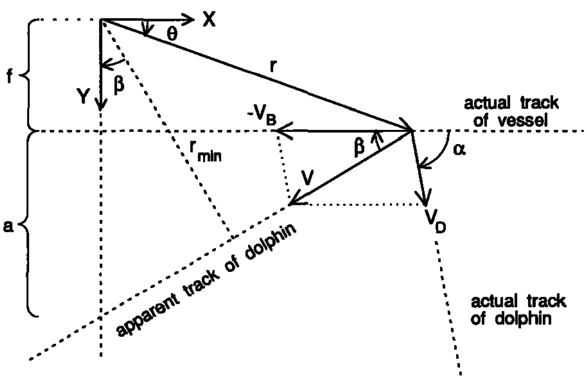

Figure 2
The apparent motion of the dolphin as seen in the moving frame of reference of the vessel. The apparent velocity **V** of the dolphin is the resultant of the vector addition of $-\mathbf{V}_B$ and $\mathbf{V}_D$. Distance f is the perpendicular distance between the initial position of the dolphin and the projected path of the vessel, while a+f is the distance between vessel and dolphin when the dolphin is abeam.

with respect to α simultaneously).

In Figure 1, c is the distance along the projected path of the vessel between the vessel's initial position and the point that is abeam of the dolphin's initial position. Let t_c be the time it takes the vessel to transverse distance c, and t that time from the initial time until the vessel has the dolphin abeam. From the application of the Pythagorean Theorem we can deduce

$$(V_D t)^2 = a^2 + [V_B(t-t_c)]^2. \quad (3)$$

Because

$$a = V_D t \sin\alpha, \quad (4)$$

it can then be shown by substitution of Eq. (4) into Eq. (3) that

$$t = \frac{t_c}{1-(V_D/V_B)\cos\alpha}. \quad (5)$$

By substituting Eq. (5) into Eq. (4), we find that as a function of α, a is given by

$$a = \frac{V_D t_c \sin\alpha}{1-(V_D/V_B)\cos\alpha}. \quad (6)$$

Because at least a is a function of α, we can conclude that in general Eq. (2) does not vanish trivially. However, β is also a function of α as can be deduced by the application of the Law of Sines to Figure 2:

$$\beta = \arctan\left[\left(\frac{V_D}{V_B}\right)\frac{\sin\alpha}{1-(V_D/V_B)\cos\alpha}\right]. \quad (7)$$

Next we investigate whether Eq. (2) vanishes nontrivially. Computing the derivative with respect to α of Eq. (6), we get

$$\frac{da}{d\alpha} = \frac{V_D t_c[\cos\alpha-(V_D/V_B)]}{[1-(V_D/V_B)\cos\alpha]^2}. \quad (8)$$

which vanishes only if

$$\alpha = \arccos\left(\frac{V_D}{V_B}\right) = \alpha_0. \quad (9)$$

Noting the similarity between a and β given respectively in Eq. (6) and (7), we can easily and simply express one in terms of the other. Writing

$$\beta = \arctan\left[\frac{a}{V_B t_c}\right], \quad (10$$

the derivative of β with respect to α is given by

$$\frac{d\beta}{d\alpha} = \frac{\cos^2\beta}{V_B t_c}\frac{da}{d\alpha}. \quad (11)$$

Substituting into Eq. (2) the equivalence of Eq. (11), we find that

$$\frac{dr_{min}}{d\alpha} = \cos\beta\left[1-\frac{(a+f)\sin\beta\cos\beta}{V_B t_c}\right]\frac{da}{d\alpha}, \quad (12)$$

which allows us to conclude that r_{min} is extreme at the point $\alpha=\alpha_0$ because, as we found in Eq. (8), a is extreme at that point. However, as can be appreciated in Eq. (12), there may be other points where r_{min} is extreme. We now investigate whether r_{min} is extreme for values of α other than α_0.

Other extreme values of r_{min} may be attained if

$$\cos\beta = 0 \quad (13)$$

or

$$(a+f)\sin\beta\cos\beta = V_B t_c \quad (14)$$

are physically realizable. Eq. (13) is satisfied for $\beta = n\pi/2$ ($n=1,3,5,\ldots$), and of these only $\beta=\pi/2$ concerns us. It corresponds to the upper limit of the physically realizable range of $0\leqslant\beta\leqslant\pi/2$, and represents the uninteresting case $V_D=V_B$ (i.e., $\alpha_0=0$) which is the trivial special case of this problem: r_{min} is rendered constant when $V_D=V_B$ (i.e., the dolphin swims with the same velocity as the vessel), so the vanishing of Eq. (2) is trivially satisfied.

In Eq. (14) let $f=0$. This limit will not diminish the generality of the solution. The direction a dolphin should take to maximize its distance to the vessel at the point of closest approach will not depend on how close the dolphin is initially to the projected path of the vessel. So without loss of generality we investigate if there is a physically realizable angle β such that

$$a\sin\beta\cos\beta = V_B t_c \quad (15)$$

is satisfied. With the result from Eq. (10) and the identity $\tan\beta=\sin\beta\cos^{-1}\beta$, Eq. (15) can be expressed as the condition $\sin\beta=\pm1$ which is satisfied by the same uninteresting case that satisfies Eq. (13). Therefore, we can conclude that the nontrivial extreme value attained by r_{min} at $\alpha=\alpha_0$ is unique in the interval $0\leqslant\alpha\leqslant\pi$.

We only have left to show that the extreme value attained by r_{min} at $\alpha=\alpha_0$ is a maximum. Let a_0 and β_0 be the respective values of a and β at $\alpha=\alpha_0$. The second derivative of r_{min} with respect to α evaluated at $\alpha=\alpha_0$ is given by

$$\frac{d^2r_{min}}{d\alpha^2}\Big|_{\alpha_0} = \cos\beta_0\left[1-\frac{a_0\sin\beta_0\cos\beta_0}{V_B t}\right]\frac{d^2a}{d\alpha^2}\Big|_{\alpha_0}. \quad (16)$$

Because the second derivative of a with respect to α evaluated at $\alpha=\alpha_0$ is

$$\frac{d^2a}{d\alpha^2}\Big|_{\alpha_0} = \frac{-V_D t_c}{[1-(V_D/V_B)^2]^{3/2}} < 0, \quad (17)$$

to determine whether Eq. (16) is negative we must show that

$$a_0\sin\beta\cos\beta_0 < V_B t_c. \quad (18)$$

We have shown already that Eq. (15) can only be satisfied for an angle $\beta=\pi/2$. Then Eq. (18) is satisfied for $0\leqslant\beta<\pi/2$. We have seen already that $\beta=\pi/2$ when $\alpha=0$, so $da/d\alpha=0$ at that point also. Therefore, we can also conclude that the nontrivial extreme value achieved by r_{min} at $\alpha=\alpha_0$, unique in the interval $0\leqslant\alpha\leqslant\pi$, is a maximum. Because Eq. (2) vanishes term by term, the same result is achieved by finding the extreme value with respect to α of either β or a.

In conclusion, a dolphin escaping at speed V_D at an angle α relative to the velocity V_B of an approaching vessel will maximize its distance to the vessel at the point of closest approach if $\alpha=\arccos(V_D/V_B)$.

Determination of dolphin velocity from range and bearing measurements

In this section we relate the dolphin velocity to practical *in situ* measurements. We will derive a relationship between dolphin speed and direction to its range $0 \leqslant r < \infty$ and bearing $0 \leqslant \theta \leqslant 2\pi$ from the vessel that triggers the dolphin to flight.

For the times $\{t_i: i=1,2,...,n\}$ we perform the corresponding measurements $\{r_i, \theta_i: i=1,2,...,n\}$. It is not necessary that the measurements be made from the vessel, but with reference to it (e.g., aerial measurements). However, it is necessary that there be no other vessel in the vicinity that perturbs the measurements by reaction of the dolphin to its presence.

The measurements of range and bearing of the dolphin from the vessel are equivalent to the cylindrical coordinates of the dolphin with respect to the moving frame of reference of the vessel. The cartesian coordinates $-\infty < x < \infty$ and $-\infty < y < \infty$ with respect to the same frame of reference are determined from

$$x = r \cos \theta \text{ and } y = r \sin \theta. \tag{19}$$

Let Δx, Δy, and Δt be, respectively, the increments of the variables x, y, and t. For each of the $(n-1)$ consecutive intervals, we can compute the average speeds in the x and y directions by

$$V_x = \frac{\Delta x}{\Delta t} \text{ and } V_y = \frac{\Delta y}{\Delta t}. \tag{20}$$

These are the components of the dolphins' apparent velocity **V** in the frame of the moving vessel (Fig. 2). We can express the dolphin velocity in the moving frame as a function of its speed and direction in the stationary frame by

$$V_x = V_D \cos \alpha - V_B \text{ and } V_y = V_D \sin \alpha, \tag{21}$$

which are a system of two coupled, nonlinear equations with V_D and α as unknowns. The solution to this set of equations is given by

$$V_D = \sqrt{V_x + V_B + V_y{}^2}, \tag{22}$$

where we have chosen the positive root, and

$$\alpha = \arctan\left(\frac{V_y}{V_x + V_B}\right). \tag{23}$$

With these results we can compare the $(n-1)$ time-intervals the direction α taken by the dolphin given Eq. (23), with the optimal direction given in Eq. (9) computed from the result given Eq. (22).

Acknowledgments

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Citations

Au, D., and W. Perryman
1982 Movement and speed of dolphin schools responding to an approaching ship. Fish. Bull., U.S. 80:371–379.

Au, D., and D. Weihs
1980 At high speeds dolphins save energy by leaping. Nature 284(5756):548–560.

Burnham, K.P., D.R. Anderson, and J.L. Laake
1980 Estimation of density from line transect sampling of biological populations. Wildl. Monogr. 72, 202 p.

Hewitt, R.P.
1985 Reaction of dolphins to a survey vessel: Effects on census data. Fish. Bull., U.S. 83:187–193.

Perrin, W.F.
1968 The porpoise and the tuna. Sea Frontiers 14:166–174.
1969 Using porpoise to catch tuna. World Fishing 18(6): 42–45.

Stuntz, W.E., and W.F Perrin
1979 Learned evasive behavior by dolphins involved in the eastern tropical Pacific tuna purse seine fishery. *In* Abstracts from presentations at the Third Biennial Conference of the Biology of Marine Mammals, Seattle, Oct. 7-11, 1979, p. 58. [Avail. Library, Natl. Mar. Mammal Lab., Seattle 98115.]