Abstract.–The accuracy and precision of estimates of catch at age from sampled lengths were evaluated for three different methods with simulated red snapper, *Lutjanus campechanus*, data for 1984–94. The methods included a growth curve, age-length keys, and a probabilistic method to classify a known total number of fish into ages from samples of the length frequency of the catch. In the first method, ages were estimated from sample lengths directly from the growth curve. The second method involved expanding the sample length frequency to age frequency by using age-length keys. The probabilistic method incorporated the cumulative frequency distributions of length at age, year-class strength, and estimates of prior survival to build age probability distributions from sampled lengths. The evaluation was based on the error in the assigned catch at age and on the resulting estimates of numbers at age and fishing mortality arising from sequential population analysis. The probabilistic method was the best of the three for the situation evaluated here, and application of the age-length key was better than that of the growth model. However, the probabilistic method requires knowledge of growth, the distributions of size at age, and recruitment that may not be known, or only poorly so. Age-length keys require no such ancillary information and may be more practical in most situations, but the probabilistic method is superior if the data requirements can be met.

Fish age determined from length: an evaluation of three methods using simulated red snapper data*

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Age-structured stock-assessment methods require estimates of the age composition of the catch. In stock assessments for Gulf of Mexico red snapper, *Lutjanus campechanus*, age compositions are used that are estimated from the sampled size distribution of the catch with growth models (Goodyear[1]). The application of age-length keys developed from age determinations of length-stratified samples of the catch is a superior method (Ketchen, 1950; Hoenig and Heisey, 1987) that has been recently incorporated into the data collection program for this stock. However, it cannot be readily applied retroactively to improve the estimates of the age composition of historical catch, and it requires significantly more resources than the former method. In this paper, I compare the precision of the estimates of the age composition of the catch from these two methods with an alternative, using simulated red snapper data. The comparisons include both accuracy and precision of the estimates of the age composition of the catch and the consequent estimates of numbers at age and fishing mortality arising from their application to sequential population analysis following the methods of Gavaris[2] and Powers and Restrepo (1992).

Methods

Simulated data

The population simulation model used in this analysis (Goodyear, 1989) employed 30 discrete ages with an instantaneous annual natural mortality (M) of 0.2 for all ages in the fishery. Each year class was further partitioned into growth platoons (cohorts with identical age but different mean lengths). The position of a growth platoon in the distribution of size at age was fixed so that the larger individuals of a year class at age 1 remained larger throughout their lifetime. Mean lengths (L) at age (A) at the beginning of January were assumed to be equal to the estimates in the 1994 stock assessment for Gulf of Mexico red snapper (Goodyear[1]) and to correspond to the von Bertalanffy equation, $L=88.24(1-\exp(-0.159$

[1] Goodyear, P. 1994. Red snapper in U.S. waters of the Gulf of Mexico. National Marine Fisheries Service, Southeast Fisheries Science Center, Miami Laboratory, Miami, Admin. rep. MIA 93/94-63, 150 p.

[2] Gavaris, S. 1988. An adaptive framework for the estimation of population size. Canadian Atlantic Fisheries Scientific Advisory Committee Research Document 88/29, 4 p.

$(A+0.458))$, where L is total length in cm and A is age in years. Size at age in the absence of fishing mortality was assumed to be normally distributed with a coefficient of variation of length at age (v) of 0.10 based on the observed variability in size at age for red snapper (Goodyear[1]). The mean length of individuals of age a in growth platoon p, l_{ap}, was determined from mean size at age (L_a) by using the normal distribution and the coefficient of variation of length at age as

$$l_{ap} = L_a + L_a z_p v,$$

where: z_p is the standard normal deviate for the pth percentile of the distribution. The simulation considered 101 growth platoons in each age class. The resulting distributions of lengths at the beginning of the year for ages 1–10 are shown in Figure 1. Within-year growth was evaluated as

$$W_{ap} = W_{a-1,p} \exp(G_{ap}),$$

where W_{ap} is the weight (kg) of an individual in growth platoon p at age a, and G_{ap} = instantaneous growth rate of growth platoon p at age a. The G_{ap} were estimated from lengths at age predicted from the von Bertalanffy growth equation.

The weight of a fish at capture W_c was evaluated as

$$W_c = W_{ap} Z_{ap} \big(\exp(G_a - Z_{ap}) - 1\big)/$$
$$\big((G_{ap} - Z_{ap})(1 - \exp(-Z_{ap})\big),$$

where Z_{ap} is the total instantaneous mortality for growth platoon p at age a during the time period. Weight was converted to length with the length-weight equation $(W=1.158 \times L^{3.056}, r^2=0.985, n=25,375)$ Growth, mortality, and catch were evaluated monthly.

The period simulated was from 1954 to 1994, but catch and sample data were retained and analyzed for 1984–94, which corresponds to the time span of actual data from the fishery. Recruitment in the model was specified by year class from 1954 to 1994 (Fig. 2). The values for 1972–94 follow the recruitment pattern observed in trawl surveys (Goodyear[1]). Earlier values were arbitrarily varied around the level observed at the beginning of the time series because landings from shrimp trawlers (predominantly juvenile fish) during these years were higher than those after 1972.

The value of fishing mortality in the model is the product of a maximum potential value for the year and a selectivity value based on the fish's age (Figs. 3 and 4). A dome-shaped selectivity schedule was
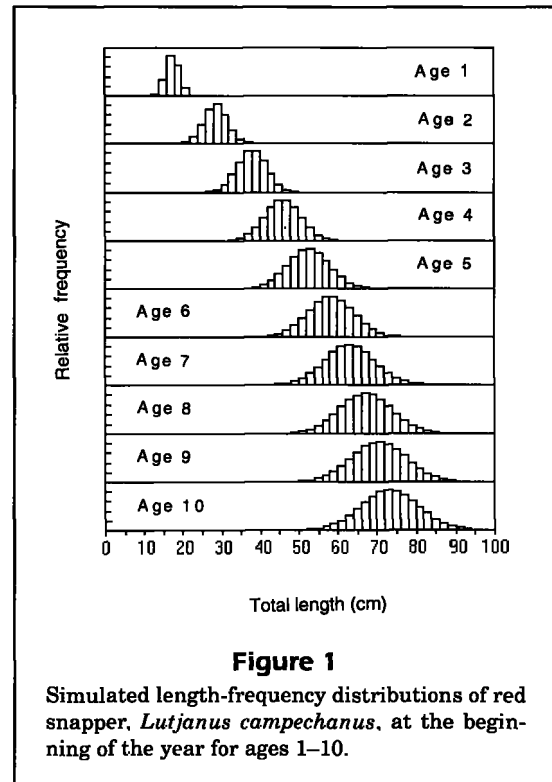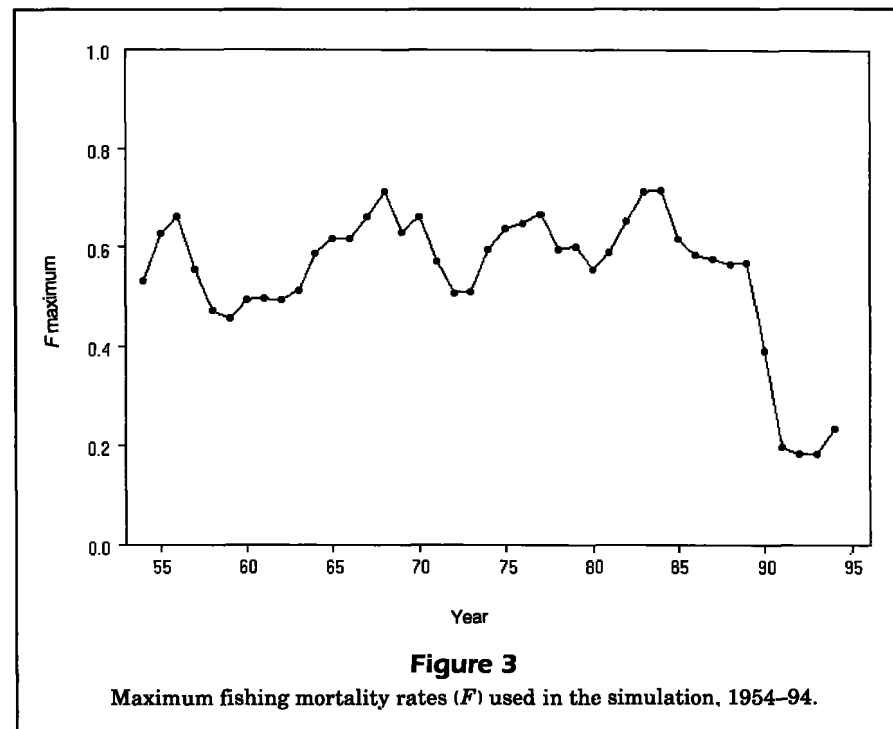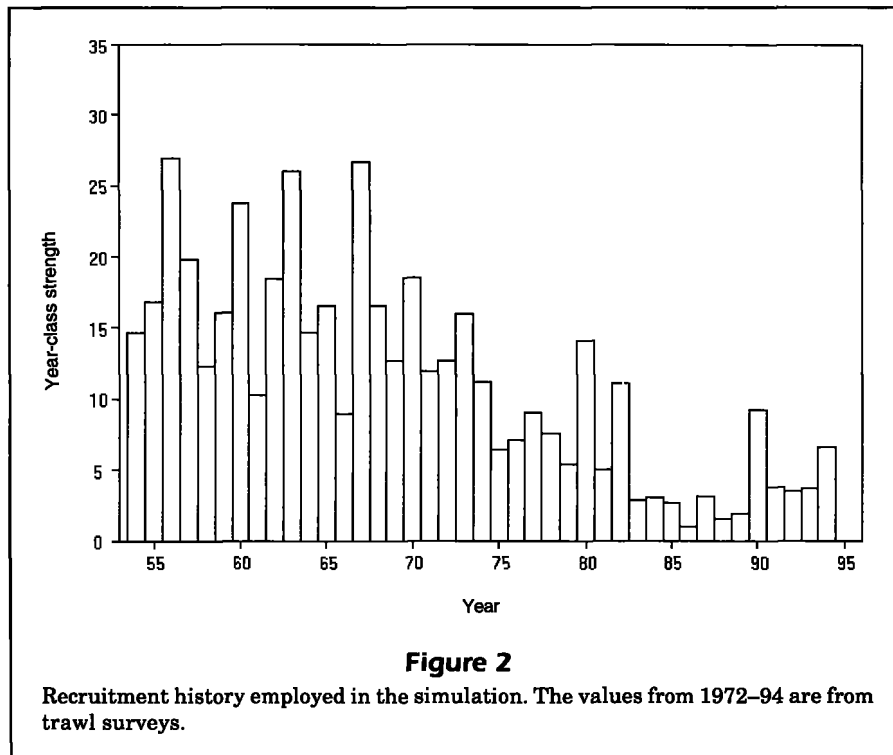

Figure 1

Simulated length-frequency distributions of red snapper, *Lutjanus campechanus*, at the beginning of the year for ages 1–10.

selected on the basis of age distribution of the catch (predominantly from handlines) in the 1994 assessment (Goodyear[1]). The value of the annual maximum for 1984–94 also follows the trend in the best estimates from the 1994 assessment, whereas earlier values were arbitrarily varied around the level observed at the beginning of the time series. The reduction in fishing mortality after 1990 was a response to management actions. The selectivity schedule (Fig. 4) was selected to produce a sample length frequency similar to that observed in the fishery (Fig. 5). Samples were truncated below 33 cm after 1990 to include the effects of changes in minimum size regulations at that time.

The simulated observations of length (and age) were obtained from the simulated catch. The catch from a growth platoon in the population structure was picked at random. It was evaluated for inclusion as an observation on the basis of the ratio of its magnitude (N_p) to the maximum catch from any other growth platoon (N_{max}). This was accomplished by drawing a uniform random number (R) between 0 and 1.0. If the ratio $N_p/N_{max} \geq R$, the length and age attributes of the cell were included as an observation; otherwise, they were discarded. This convention caused the sampled growth platoon to be proportional to their magnitude in the simulated catch. The process was repeated for each month of the simulation until 1,000 samples had been drawn. This provided 12,000 length

Figure 2

Recruitment history employed in the simulation. The values from 1972–94 are from trawl surveys.

Figure 3

Maximum fishing mortality rates (*F*) used in the simulation, 1954–94.

samples per year. No error was added to either age or length to simulate measurement error.

The ages of the first two fish sampled in each 1-cm length stratum each month were retained with their lengths to build the age-length key for that month.

This provided a maximum sample size of 24 fish per length stratum or about 4,000 fish per year to construct the age-length key, but sample size varied slightly because of the stochastic nature of the sampling process.
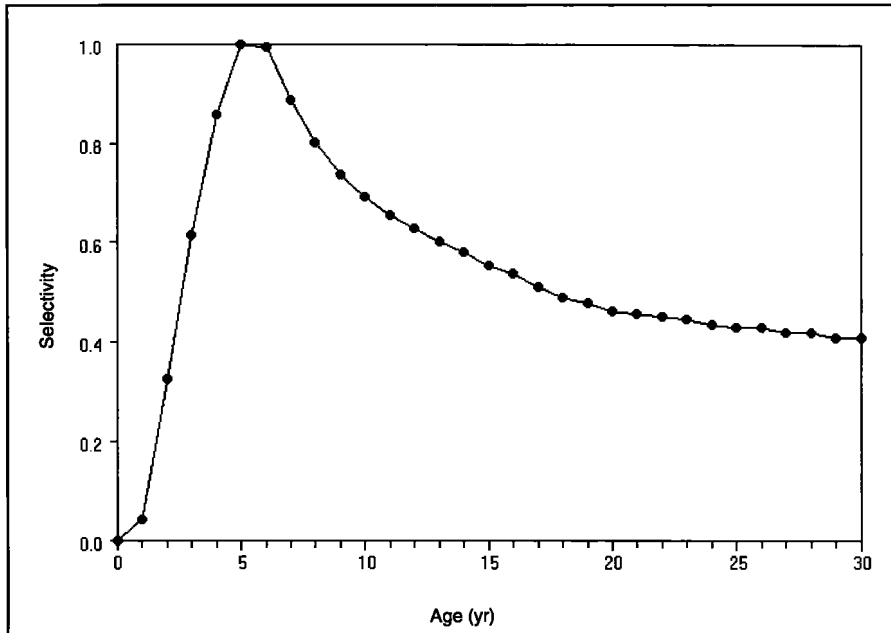
Figure 4

Selectivity schedule used in the simulation. The fishing mortality rate is the product of the maximum fishing mortality rate (*F*) and the selectivity value corresponding to fish age.
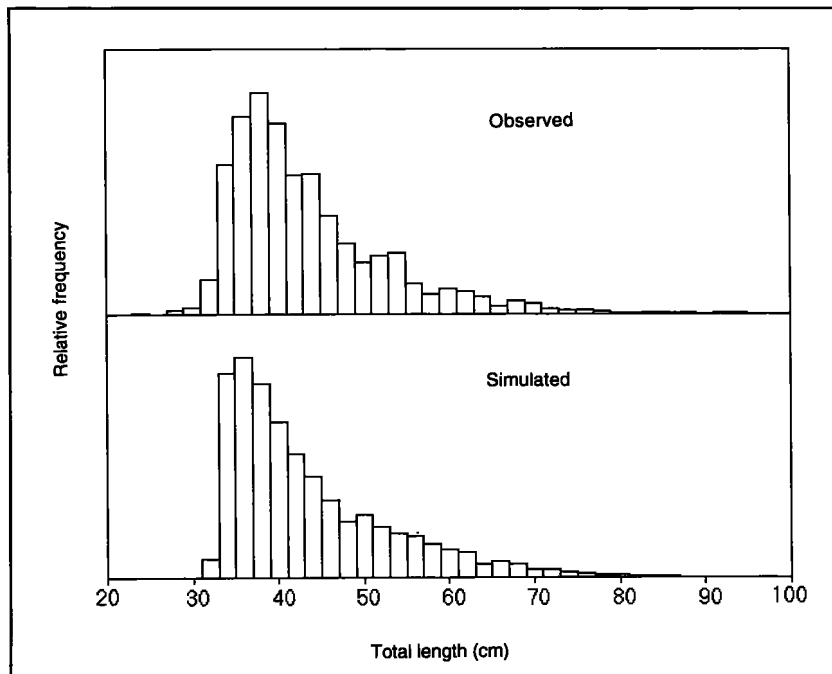

Figure 5

Observed and simulated length frequencies of red snapper harvested in 1993.

Age-estimation methods

For all three ageing methods evaluated, the number of fish in the catch at age a, N_a, was estimated as

$$N_a = Cf_a,$$

where C, the total catch in numbers of fish, was the known value from the simulation, and f_a was the estimated fraction of catch at age a. Age frequencies were estimated separately for each year between 1984 and 1994.

With the first method, the von-Bertalanffy growth equation was rearranged to predict age from length,

$$A = -\log_e(1 - L/88.24)/0.159 - 0.458,$$

and the f_a were estimated as the ratios of the number of sampled fish assigned age A to the total number of fish in the sample. For this method any sampled fish larger than the asympotic size was discarded.

With the second method typical age-length keys (Ketchen, 1950; Westrheim and Ricker, 1978) were constructed annually from the monthly age-frequency samples. In this case the f_a were estimated by multiplying the observed age frequencies for each length stratum by the ratio of length samples in each length stratum to the total number of length samples and by summing over ages.

The third (probabilistic) method is a proposed alternative and requires estimates of prior survival of year classes in the population and independent estimates of year-class strength. In this method

$$f_a = \frac{\sum\limits_{i=1}^{j}\sum\limits_{a=0}^{n} P_a}{j},$$

where j is the number of length samples, n is the number of ages, and

$$P_a = \frac{W_a R_{y-a} S_a}{\sum\limits_{a=1}^{n} W_a R_{y-a} S_a}$$

and

$$W_a = \frac{dD_a}{dL_i},$$

where D_a is the cumulative probability distribution of length for age a, L_i is the observed length of fish i, R_{y-a} is the recruitment strength in year $y-a$, y is the year of observation, and S_a is survival probability from recruitment to age a and is given by

$$S_a = \exp\sum_{i=0}^{a-1} -(F_i + M_i),$$

where F_i is the fishing mortality of the year class at age a when it was age i, and M_i is the natural mortality of the year class at age a when it was age i.

Inspection of the data used in this method reveals that the method requires values for nearly everything one would wish to estimate from the age composition of the catch and consequently seems to place the cart before the horse. However, in many cases ancillary data on year-class strength may be available from research surveys, and estimates of natural and fishing mortalities may be available from earlier assessments. In this investigation, this method was applied in two ways. The first assumed preexisting accurate knowledge of year-class strengths and mortality. The second application assumed knowledge of year-class strengths and natural mortality and proceeded iteratively. In the first iteration, age composition was estimated with the assumption that there was no fishing mortality. This led to a set of estimates of catch at age that were then used through sequential population analysis to estimate fishing mortality at age. With the second iteration the resulting estimates of fishing mortality were added and catch at age was reestimated. This process was repeated several additional times.

Overall, the three methods provided 4 sets of estimates of catch at age that could be compared with the true values from the simulation: those from the growth model, those from the age-length key, those from the probabilistic method given knowledge of survival, and those from the iterated probabilistic method. In addition, numbers at age and fishing mortality for each year were estimated from the catches at age for each set by using sequential population analysis (Powers and Restrepo, 1992). For the purpose of this exercise, the selectivities for the terminal year of the population analysis were the known values from the simulation, and the tuning index was the known number of age-4 individuals alive at the beginning of the year. The methods were compared by correlating the known true values from the simulation to the values estimated by each method. Because there were 31 ages in the model (0–30) and 11 years, these provided a total maximum sample size of 341; however, year-age combinations where the true catch at age was below 100 were dropped. Thus sample sizes for most analyses were reduced to 331. In addition, scattergrams of the logs of the ratios of estimated to true values were constructed for each comparison. The r^2 values for the correlations between true and estimated values are presented with

each of the scattergrams. Although the scattergrams involve transformations to reflect the error more accurately, the correlations themselves are based on the untransformed data.

Results

The estimates of catch at age from each of the methods were highly correlated with the true values (Fig. 6). But the error in catch at age was clearly highest for the ages assigned with the growth model (Fig. 6A). Catch at age from the age-length key was considerably better than that from the growth model, particularly for the younger more abundant ages in the catch (Fig. 6B). The younger ages in these figures tend to be to the right side of the scattergrams and the older, less abundant ages are on the left.

The probabilistic method, given prior knowledge of fishing mortality and recruitment, provided the best result, with very little difference between true and estimated age compositions except at the oldest ages (Fig. 6C). The bias in the estimates obtained with this method with only natural mortality is evident in Figure 6D, but even so, the estimates for the youngest ages are better than the estimates from the growth model. The bias was reduced by the fifth it-

eration (Fig. 6E) and almost completely removed by the tenth iteration (Fig. 6F).

The estimates of number at age derived from each set of catch at age by using an age-sequenced analysis are presented in Figure 7. Again the results were least favorable for the catch-at-age matrix developed from the growth model (Fig. 7A), followed by the age-length key (Fig. 7B) and the probabilistic method (Fig. 7C). The bias in estimated number at age from the probabilistic method, where fishing mortality is not used, is even more pronounced than it was for the catch-at-age matrix (Figs. 5D and 6D). However, the bias was reduced by the fifth iteration by using the fishing mortality rates derived from prior iterations and almost completely removed by the tenth iteration (Fig. 7, E–F). The similarity of r^2 values for the correlations between observed and predicted values for the age-length key and probabilistic methods in Figures 6 and 7 are somewhat misleading because of the very large dynamic range of the numbers and corresponding catches at age used in the analysis. In actuality, the precision of the estimates arising from applicaton of the age-length key was much lower than that for the probabilistic method for age classes that were infrequent in the catch.

The estimates of fishing mortality at age, derived from each set of catch at age by using age-sequenced
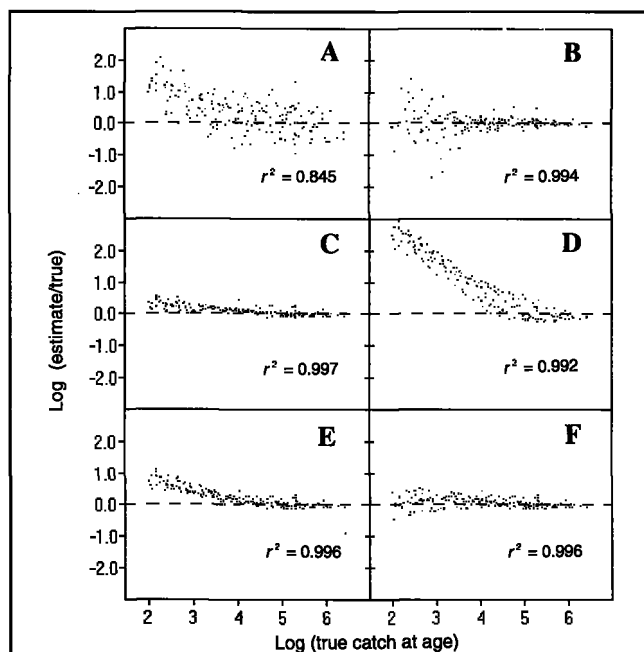

Figure 6

Ratios of estimated to true catch at age from the growth model (A), age-length key (B), and from the probabilistic method with knowledge of prior survival (C), and probabilistic iterations 1, 5 and 10 (D–F).
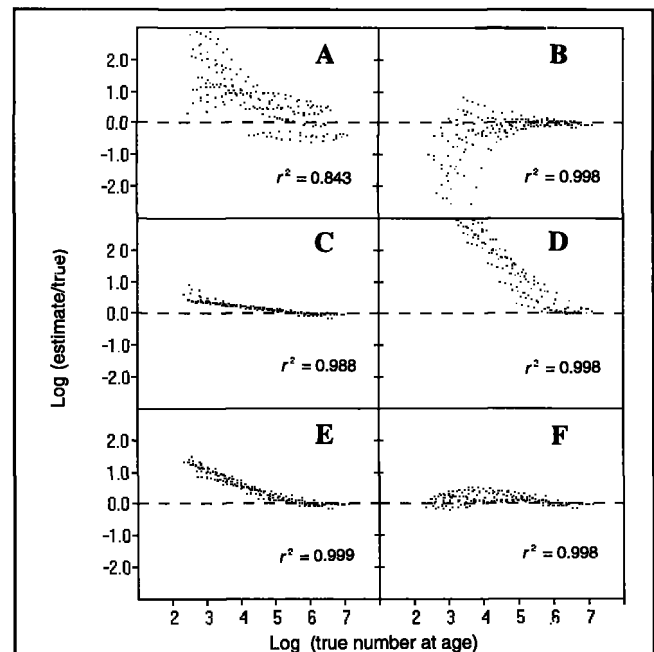

Figure 7

Ratios of estimated to true number at age from analysis of catch at age from the growth model (A), age–length key (B), probabilistic method with knowledge of prior survival (C), and probabilistic iterations 1, 5 and 10 (D–F).

analysis, are presented in Figure 8 for ages up to 10. Again, the results were least favorable for the catch-at-age matrix developed from the growth model (Fig. 8A), followed by the age-length key (Fig. 8B), and the probabilistic method (Fig. 8C). The upward bias in estimated number at age from the probabilistic method in the absence of fishing mortality in Figure 7D led to an underestimate of fishing mortality of Figure 8D. However, the bias was reduced by the fifth iteration and almost completely removed by the tenth iteration (Fig. 8, E–F).

The relatively higher error in the catch at age for older ages estimated by using the growth model and age-length key (Fig. 6, A–B) led to relatively higher error in the estimates of numbers at age from their analysis. This resulted in poor estimation of fishing mortality for the oldest ages in the simulated catch which caused the correlation between true and estimated fishing mortalities to decline when fish older than 10 years were included in the analysis (Fig. 9, A–B). The results from the probabilistic approach also showed a similar trend but were much less sensitive than those for the other two methods (Fig. 9, D–F).

Discussion

These results indicate that for the situation evaluated here the probabilistic method is superior to age assignment from either a growth model or an age-length key. Factors leading to this conclusion include knowledge of the actual history of year-class strengths and perfect knowledge of growth, natural mortality, and the distribution of size at age. Imperfect knowledge of any of these elements would degrade the performance of the probabilistic method. If there are sufficient data to develop a growth curve then it should be possible to characterize the distribution of size at age, at least for the more abundant ages in the population. Poor knowledge of the growth curve itself would also adversely affect the estimates obtained directly from the growth curve.

The results from the age-length key would be unaffected by poor knowledge of growth, past recruitment, and natural mortality. However, the comparisons among methods in the current analysis assumed no error in age assignments for the age-length key. Experience suggests that there is uncertainty in age assignment from hard-part analysis, an uncertainty that increases with fish age (Beamish and Fournier, 1981). Including such error would have added to the difference between the results of this method and those obtained with the probabilistic method. Nonetheless, the construction and application of age-
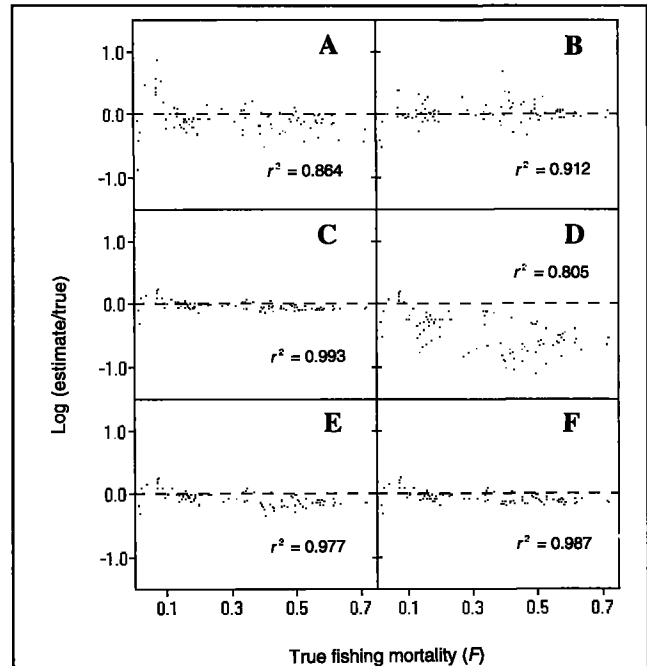

Figure 8

Ratios of estimated to true fishing mortality (F) from analysis of catch at age from the growth model (A), age-length key (B), probabilistic method with knowledge of prior survival (C), and probabilistic iterations 1, 5, and 10 (D–F).
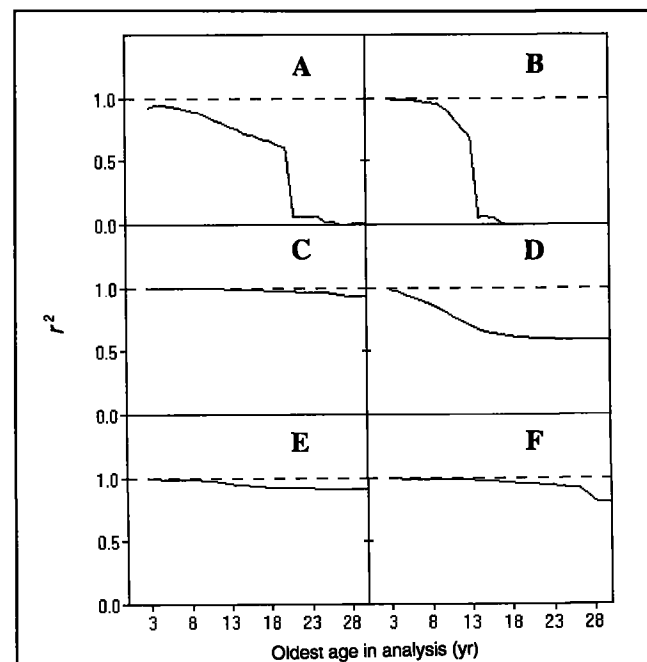

Figure 9

Precision of fishing mortality estimates (r^2 from correlations of true and estimated rates) from the growth model (A), age-length key (B), probabilistic method with knowledge of prior survival (C), and probabilistic iterations 1, 5, and 10 (D–F).

length keys involves the fewest assumptions. Where almost certain knowledge of growth and year-class strengths is lacking, and the method for ageing the fish is robust, this method is probably the best choice for monitoring the age composition of a catch.

Where time-series data for year-class strengths are available, where growth and natural mortality are reasonably known, and where there are insufficient age determinations to construct age-length keys, the probabilistic method is clearly superior to age assignments from inverted growth models and also might be as good as, or better than, the age-length keys if they were available. Where growth and year-class strengths are well characterized and natural mortality is reasonably known, the probabilistic method should outperform all the other alternatives. Additionally, this method should be very useful for estimating the age composition of catches for the most recent year of a time series for which sample-age analysis may not yet be complete. It should also provide a reliable method to estimate the age compositions of catches for intermediate years of a time series, for which insufficient age determinations are available to construct age-length keys.

Acknowledgments

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