BIAS AND VARIANCE IN ALLEN'S RECRUITMENT RATE METHOD

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ABSTRACT

The method of estimation of the recruitment rate of a population based upon the methodology of Allen is reviewed, and a simpler formulation is presented. The estimator is evaluated for bias and variability. If the recruitment pattern at age is constant with time, the technique shows no bias provided the age of first full recruitment is not underestimated. Use of age-length keys will tend to spread partially recruited ages upwards so the age of first full recruitment should accommodate this. An approximate analytical formula for the variance of the estimated recruitment rate is given, and this shows that variance decreases with increasing second year catches.

If recruitment to a fishery changes such that it occurs at an earlier age with time, then high values of recruitment rate are given and vice versa. This could be interpreted erroneously as an increasing or decreasing population growth rate, if it was assumed that the pattern of new recruitments at age had been constant. It is also found that a fluctuating recruitment pattern will give a negatively biased rate. These last considerations suggest that the method should not be used unless a constant recruitment pattern can be established. For a series of years of data other techniques should be used.

Allen (1966, 1968) introduced a technique for the estimation of the proportion of new recruits to total recruits in an exploited stock. This statistic is particularly useful since it is necessary for the simulation of stock dynamics and hence it is used in the estimation of stock size. As proposed by Allen the statistic is obtained directly from the catch of the previous 2 years, and this is in contrast to estimates obtained by virtual population analysis, which requires several years of data for a comparable estimate to converge to a satisfactory answer. Allen termed this parameter r_{II} but here it will be denoted by r. The only data needed to calculate this recruitment rate are the proportions of catch at age for 2 consecutive years, and a knowledge of the age of first full recruitment. Allen (1973) used this method to calculate the recruitment rates of fin whales in the Antarctic and so constructed a stock and recruit relationship and estimated stock sizes. The use of Allen's method of obtaining recruitment rates has been advocated by Ricker (1975) and Gulland (1977: Chapter 1 by Ricker, Chapter 4 by Gulland, Chapter 14 by Allen and Chapman). The method has been used extensively in stock assessments by the International Whaling Commission but often has given average values that were thought to be unreasonably low. This can be seen for sei whales in Ohsumi (1978) and for minke whales in Chapman (1983). In addition, the rates have been very variable even with moderately high catches (Ohsumi 1978; Allen 1982). Some properties of the estimate have been considered. Ricker (1975) investigated the effect of changes in some of the population parameters and found a negative bias if the first age of full recruitment was underestimated, and Allen (1981) looked at the sensitivity of the method to variable catches at age and concluded that although variability of the estimate was high for catches less than a few hundred, the bias was small even for low catches. This still left unresolved the question of why the estimated recruitment rates were often so low and why they were more variable than Allen's (1981) simulations predicted. Ohsumi's results gave coefficients of variation over 200%, whereas Allen's simulations gave about 75% if catches were low.

This study investigates the behavior of the estimated recruitment rate from Allen's model. The estimator is fully introduced in order to demonstrate that the existence of a free and selectable parameter (Allen's T) is erroneous; once established the estimator then reduces to a simple form. A typical age-structured model is introduced and the effects on the estimated recruitment rate of changes from year to year of demographic parameters of the model are investigated. The effect of sampling variability in the number of animals caught at age is reviewed in relation to bias and variance of the estimate, and an expression for the approximate analytical variance of the recruitment rate is pre-

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sented. However the most important finding is that it is shown that trends in recruitment rate and changes in selection pattern are confounded.

DERIVATION OF RECRUITMENT RATE ESTIMATOR

Allen's (1966, 1968) definition of recruitment rate is the proportion of new recruits to total recruited stock, that is, the number of animals newly available for capture compared with the total available in that year. It is explicitly defined through Equation (1) below. Retaining the same nomenclature as Allen (1966) let us define the following terms:

$N_{i,t}$: numbers in the population of age i in year t,
$C_{i,t}$: catch in numbers of age i in year t,
$U_{i,t}$: proportion of age group i in year t that are exploitable, i.e., recruited,
$P_{i,t}$: proportion of the total catch from age i in year t,
$Q_{i,t}$: proportion in the catch of year t from all fully recruited age groups $\geqslant i$,
CT_t : total catch in year t,
M : instantaneous natural mortality rate,
F_t^a : instantaneous fishing mortality rate on fully recruited ages in year t, ages $i \geqslant k$,
Z_t^a : instantaneous total mortality rate, $F_t^a + M$, on fully recruited ages,
F_t^j, Z_t^j : instantaneous fishing and total mortality rates on pre-fully recruited ages, ages $i < k$,
F_t, Z_t : instantaneous mortality rates if there are no differences between juvenile and adult rates,
k : age of first full recruitment, $U_{i,t} = 1.0$ $i \geqslant k$,
r_t : net recruitment rate in year t.

Let us define $\phi_t^j = F_t^j(1.0 - \exp(-Z_t^j))/Z_t^j$ and similarly for ϕ_t^a. By definition, the recruitment rate is

$$r_2 = \frac{\text{new recruits}}{\text{total exploitable}} = \frac{\sum_{i=x}^{k} N_{i,2}\, U_{i,2} - (\exp(-Z_1^j)) \sum_{i=x}^{k-1} N_{i,1}\, U_{i,1}}{\sum_{i=x}^{\infty} N_{i,2}\, U_2}, \qquad (1)$$

where x is the age of first partial recruitment. This can be rewritten as

$$r_2 = \frac{\sum_{i=x}^{k-1} C_{i,2}/\phi_2^j + C_{k,2}/\phi_2^a - (\exp(-Z_1^j)) \sum_{i=x}^{k-1} C_{i,1}/\phi_1^j}{\sum_{i=x}^{k-1} C_{i,2}/\phi_2^j + \sum_{i=k}^{\infty} C_{i,2}/\phi_2^a}.$$

Multiplying top and bottom by ϕ_2^j/CT_2 gives

$$r_2 = \frac{\sum_{i=x}^{k-1} P_{i,2} + P_k\, \phi_2^j/\phi_2^a - (\exp(-Z_1^j)) \sum_{i=x}^{k-1} C_{i,1}\, \phi_2^j/\phi_1^j\, CT_2}{\sum_{i=x}^{k-1} P_{i,2} + (\phi_2^j/\phi_2^a) \sum_{i=k}^{\infty} P_{i,2}}.$$

Note that $\dfrac{Q_{k+1,2}}{Q_{k,1}} = \dfrac{(\sum_{i=k+1}^{\infty} C_{i,2})\, CT_1}{(\sum_{i=k}^{\infty} C_{i,1})\, CT_2} = \dfrac{CT_1\, \phi_2^a \exp(-Z_1^a)}{CT_2\, \phi_1^a}.$

Therefore

$$r_2 = \frac{\sum_{i=x}^{k-1} P_{i,2} + P_{k,2}\,\phi_2^j/\phi_2^a - Q_{k+1,2}\,(\sum_{i=x}^{k-1} P_{i,1})\,\phi_2^j\phi_1^a \exp(Z_1^a - Z_1^j)/(Q_{k,1}\,\phi_1^j\phi_2^a)}{\sum_{i=x}^{k-1} P_{i,2} + (\phi_2^j/\phi_2^a)\sum_{i=k}^{\infty} P_{i,2}}. \quad (2)$$

Compare this result with that given by Allen (1966, 1968). He gives

$$r_2 = P_{1,2} - \sum_{i=1}^{k-1} (1.0 - T_1/B_{i,1})\cdot P_{i+1,2},$$

where $B_{i,1} = P_{i+1,2}\cdot Q_{k,1}/(P_{i,1}\cdot Q_{k+1,2})$

$$T_t = \exp(Z_t^a - Z_t^j).$$

Consequently,

$$r_2 = \sum_{i=x}^{k} P_{i,2} - T_1(Q_{k+1,2}/Q_{k,1})\sum_{i=x}^{k-1} P_{i,1}. \quad (3)$$

To satisfy the above Equations (2) and (3) it is found that $\phi_2^j/\phi_2^a \equiv 1.0$. That is, the proportion fished in the two recruited age groups in the second year must be the same. If it is then assumed that the natural mortality rate, M, is the same in each group, then it is necessary that $F_2^j \equiv F_2^a$. From Equation (2) we then have

$$r_2 = \sum_{i=x}^{k} P_{i,2} - T_1(\sum_{i=x}^{k-1} P_{i,1})(Q_{k+1,2}/Q_{k,1})\phi_1^a/\phi_1^j.$$

Consequently to satisfy Allen's model, Equation (3), it is also necessary that $\phi_1^a/\phi_1^j \equiv 1.0$, and as above this implies $F_1^j \equiv F_1^a$. Hence $Z_1^a = Z_1^j$ and $T_1 = \exp(Z_1^a - Z_1^j)$ is necessarily unity, and there is no flexibility in the choice of T as Allen (1966, 1968) and Ricker (1985) suggest.

If one then assumes that, within each year, the mortality rate of all recruits is similar, then Equation (2) reduces to a very simple form

$$r_{t+1} = \alpha - \beta(1 - \alpha)/(1 - \beta),$$

where $\alpha = \sum_{i=1}^{k} P_{i,t+1}$

$$\beta = \sum_{i=1}^{k-1} P_{i,t}. \quad (4)$$

DEVELOPMENT OF A VALIDATION PROCEDURE

In order to test the robustness of the estimator given by Equation (4) we need a population model. If in year 1 the population is assumed to have a stable age structure and has been increasing at a rate of λ per year, such that the population vector,

$$N_{t+1} = \lambda N_t,$$

and where the mortality rate, Z, has been constant over time, then with an arbitrary number of 1-yr olds the numbers at age in year 1 can be calculated from the recurrence relationship:

$$N_{1,1} = 5{,}000.$$

$$N_{i+1,1} = N_{i,1}[U_{i,1}\exp(-Z_1) + (1.0 - U_{i,1})\exp(-M)]/\lambda,$$

$$N_{21,1} = N_{20,1}\cdot\exp(-Z_1)/\{\lambda(1.0 - \exp(-Z_1))\}.$$

It is assumed that $U_{20,1} = 1.0$, i.e., $k \leqslant 20$ and N_{21} is a "plus-group" of ages $\geqslant 21$. The age subscripts have now been dropped from Z and F. Consequently the numbers in the second year are given by

$$N_{1,2} = \lambda\cdot 5{,}000.$$

$$N_{i+1,2} = N_{i,1}[U_{i,1}\exp(-Z_1) + (1.0 - U_{i,1})\exp(-M)]$$

$$N_{21,2} = (N_{20,1} + N_{21,1})\exp(-Z_1)$$

$$P_{i,t} = (N_{i,t}\,U_{i,t})/\sum_{i=1}^{21}(N_{i,t}\,U_{i,t}). \quad (5)$$

If we wish to consider the effects of stochastic catches at age or problems in aging, then $P_{i,t}$ becomes a variable, $\bar{P}_{i,t}$, and can be expressed in terms of the catch, so that

$$\bar{P}_{i,t} = C_{i,t}/CT_t,$$

where $C_{i,t}$ is determined as an independent random variable. For the expected catch at age, $m \geqslant 50$ $C_{i,t}$ is distributed as $N[m_{i,t}, m_{i,t}(1 - P_{i,t})]$ where $P_{i,t}$ is calculated from Equation (5), and for $m < 50$ as Poisson $[m_{i,t}]$. m is obtained as

$$m_{i,t} = F_t(1.0 - \exp(-Z_t))\, N_{i,t}\, U_{i,t} / Z_t,$$

and

$$CT_t = \sum C_{i,t}.$$

If aging of the catch introduces bias or variance this can be investigated using a matrix A where the element $a_{i,j}$ is the probability that an animal of true age i will be called j. The new catch at allocated age can be given by C' where

$$C'_t = A\, C_t,$$

and where C is the column vector of catch at age.

From the validation model the true recruitment rate can be calculated as

$$\frac{N_{1,2}U_{1,2} + \sum_{i=1}^{k-1} (N_{i+1,2}U_{i+1,2} - N_{i,1}U_{i,1} \exp(-Z_1))}{\sum_{i=1}^{21} N_{i,2}U_{i,2}} \qquad (6)$$

It can be easily shown that, for $U_{i,1} = U_{i,2}\, N(t = 1)$ in a stationary age composition, this reduces to

$$(\lambda - \exp(-Z_1))/\lambda. \qquad (7)$$

In the following tests the results using Equations (6) and (4) will be compared when parameters are changed from year to year or when variability is introduced. In all tests $F_1 = 0.05$ and $M = 0.05$.

RESULTS

The results of comparing the true recruitment rate from Equation (6) with those obtained from Equation (4) are given below, for the cases when the fishing mortality is different in 2 adjacent years, for $\lambda \neq 1$, and for the age at first full recruitment (k) incorrectly chosen when $U_{i,2} = U_{i,1}$ (sections i - iii below). Section iv considers the effect of variability and biases in the age determination of the catch and section v considers stochastic effects, all with $U_{i,2} = U_{i,1}$. Section vi considers the effects of $U_{i,2} \neq U_{i,1}$.

(i) $F_2 \neq F_1$

From Equation (6) it is evident that the value of F_2 ($F_2 \neq 0$) does not affect the recruitment rate and this is reflected in Equation (4) where only proportions in the catch each year are needed. The value does however affect the variance of r as shown later. This was also noted by Ricker (1975).

(ii) $\lambda \neq 1.0$

Equation (7) shows that r is a funtion of λ, the rate of increase of the population given F_1. Equation (4) accurately gives the true value of r irrespective of λ or F_2.

(iii) k - Incorrectly Chosen

Let k' be the selected age at first full recruitment. If $k' \geqslant k$ then Equation (4) gives the same rate as Equation (6). As a confirmation of Ricker's (1975) findings it is easy to show that in the extreme case of knife-edge recruitment k' can be $\geqslant k$ and that if k' is $< k$ then no new recruitment is detected. For $U_i = 0.1i$, $i = 1$ to 10 and $U_i = 1.0$, otherwise Table 1 shows the reduction in r using Equation (4) as k' is reduced from its true value at $k = 10$. The reduction is substantial and the effect on the estimated net recruitment rate, r', is more so. r' is calculated as (Chapman 1983)

$$r' = r - 1 + \exp(-M).$$

It can be seen that in this example, which is not unlike many examples in whale assessments, an error of 1 year would reduce r' by 20%. (It is worth noting that this equation for the net recruitment rate is approximate and underestimates the true net rate by about the product of F and M or F and Z depending

TABLE 1.—Reduction in recruitment rates (r) as k' is incorrectly chosen $<k = 10$, and net recruitment rate, r'.

k'	10	9	8	7	5	3
r	0.095	0.086	0.077	0.068	0.051	0.032
r'	0.046	0.037	0.028	0.022	0.002	−0.016

on when the catches are removed from the population.)

(iv) Aging of Catch Biased

If aging is biased such that each age is wrongly allocated to another specific age then the matrix A may look like Table 2(a), which would indicate that all animals age i were called $i + 1$ for $i = 1$ to 4. If the true age of first full recruitment, k, is 3 and this was used in Equation (4) an underestimate would result as described above. This may occur if k is obtained independently of the catch data. If however the catch data are used to estimate k this will also be aged incorrectly with the result that the estimate of r is correct. In the example $k' = 4$ and this value used in Equation (4) yields the correct answer.

TABLE 2.—Matrix A of allocated age against true age.

(a)	(0	0	0	0	0)	(b)	(1	0.2	0	0	0)
	(1	0	0	0	0)		(0	0.6	0.2	0	0)
	(0	1	0	0	0)		(0	0.2	0.6	0.2	0)
	(0	0	1	0	0)		(0	0	0.2	0.6	0.2)
	(0	0	0	1	1)		(0	0	0	0.2	0.8)

Often an age-length key is used to age the catch. This implies that an animal of true age i will be allocated to age j with a probability distribution centered upon i. Table 2(b) shows matrix A in such a case. If, in this example, $k = 3$, the spreading of some of the catch at age 3 into age 4 means a bias will result from the omission of this group. Consequently the k used in Equation (4) needs to be 4 to avoid a negative bias. The value of k used needs to be such as to ensure that all partly recruited ages are counted and if age-length keys are used, it may have to be substantially higher than the true age of first full recruitment.

(v) Stochastic Results

In this section we consider the biases introduced by variability occurring in the system.

Variance in Catch at Age

Variance in catch at age in the first year was modelled with either a Poisson or Normal distribution as previously described. For the second year the age distribuion was found given these stochastic catches and an expected catch in year 2 was obtained given F_2. From the expected $C_{i,2}$ a second stochastic set of catches was obtained. This was repeated for 50 pairs of years with $U_i = 0.2(5 - i)$ for i from 6 to 10 and 1.0 for older ages.

For a range of λ and F_2 the average value of r, calculated from Equation (4), was accurate to within a few percent either way of the true value of r. This is in agreement with the findings of Allen (1981). However, even for small values of F_1 and F_2, and hence low catches, there was no evidence of a general bias; this is contrary to the findings of Allen (1981).

Variance of r

From the population model the variance of r can be calculated for given F_1 and F_2. For each of the 50 simulations described above the mean and variance of r was calculated. The coefficients of variation agreed well with those described by Allen (1981, table 1) ranging from 0.15 for a second year catch of 2,500 to 0.76 for a catch of 75.

A theoretical variance is derived below.

$$\text{Let us define } \theta_t = Z_t/\{F_t(1.0 - \exp(-Z_t))\}$$

$$\text{and } \theta = \theta_1/\theta_2.$$

Let x = the age of first partial recruitment, remember $U_{i,1} = U_{i,2}$, and CT_t is the total catch in year t. Then from Equation (6) it can be seen that r can be written as

$$r_2 = (\sum_{i=1}^{k} C_{i,2} - \theta \exp(-Z_1) \sum_{i=1}^{k-1} C_{i,1})/CT_2.$$

$$\text{Note that } E(C_{i,2}) = N_{i,2}U_i/\theta_2 \qquad i = x + 1 \text{ to } 20$$

$$= \zeta_i \, C_{i-1,1}$$

where $\zeta_i = \theta(U_{i-1} \exp(-Z_1) + (1.0 - U_{i-1}) \exp(-M)) \, U_i/U_{i-1}$, and writing the catch from the fully recruited plus group at time 2 in terms of that at time 1 we get

$$E(C_{21,2}) = \zeta_{21} \, C_{20,1} + \zeta_{22} \, C_{21,1},$$

where $\zeta_{22} = \theta \exp(-Z_1)$.

No prior information is known about $N_{i,1}$ and so we must assume that $E(C_{i,1}) = C_{i,1}$. If $C_{i,1}$ is assumed to be a Poisson variable with parameter $(C_{i,1})$ the sample catch then $C_{i,2}$ can be assumed to be a compound-Poisson variable with a relationship that can be approximated by

$$C_{i,2} = \zeta_i C_{i-1,1} + \varepsilon_i, \qquad i = x + 1 \text{ to } 20,$$

and

$$C_{21,2} = \zeta_{21} C_{20,1} + \zeta_{22} C_{21,1} + \varepsilon_{21} + \varepsilon_{22},$$

where $E(\varepsilon_i) = 0$ and $\text{var}(\varepsilon_i) = (\zeta_i + \zeta_i^2) C_{i-1,1}$.

Consequently,

$$r_2 = [C_{x,2} + \sum_{i=x}^{k-1} (\zeta_{i+1} - \theta \exp(-Z_1)) C_{i,1} + \sum_{i=x+1}^{k} \varepsilon_i]/$$

$$[C_{x,2} + \sum_{i=x}^{21} \zeta_{i+1} C_{i,1} + \sum_{i=x+1}^{22} \varepsilon_i].$$

For any given vector N_1 and fixed Z_1 and Z_2 the above terms are independent and the approximate variance can be given by

$$\text{var}(r_2) = \text{var}(C_{x,2})(\partial r/\partial C_{x,2})^2 + \sum_{i=x}^{21} \text{var}(C_{i,1})(\partial r/\partial C_{i,1})^2 + \sum_{i=x+1}^{21} \text{var}(\varepsilon_i)(\partial r/\partial \varepsilon_i)^2$$

$$= [C_{x,2}(1 - r)^2 + \sum_{i=x}^{k-1} \{\zeta_{i+1}(1 - r) - \theta \exp(-Z_1)\}^2 C_{i,1} + \sum_{i=x}^{k-1} (1 - r)^2 (\zeta_{i+1} + \zeta_{i+1}^2) C_{i,1} + \sum_{i=k}^{21} r^2 (\zeta_{i+1} + 2\zeta_{i+1}^2) C_{i,1}]/(CT_2)^2. \quad (8)$$

Comparison of the simulated variances, with the above pattern of recruitment and $k = 10$, with those predicted from Equation (8) showed the analytical variance to be a very good approximation, averaging about 0.96 times the simulated variances. However, with $k = 12$ the analytical variance was only 0.70 times the simulated variance. It is clear from Equation (8) that the variance will decrease as the square of the second year catch increases but the first year catches play a more linear role, except through the interactions of ζ and θ with the first year catch.

Equation (8) also shows there is a cost involved with increasing k. This is desirable to avoid any bias, but if too few age classes are considered to be fully recruited, then variance increases. In the example considered, raising k from 10 to 12 yr increased variance by 40% and the simulated variances show the increase may even be greater.

Additional simulations also revealed that the use of an age-length key might reduce variance by smoothing out real differences in catch at age, but the reduction was nullified by the additional variance due to increasing k.

(vi) $U_{i,2} \neq U_{i,1}$

Equation (6) still allows a true recruitment rate to be calculated in this case and Allen's (1966) derivation allows $U_{i,2} \neq U_{i,1}$. As k should not be underestimated when used in Equation (4), then k can be defined as the larger of the two ages of first full recruitment in years 1 and 2.

In this trial an initial stable age distribution was prescribed with $\lambda = 1.0$, $F_1 = 0.05$ and

$$\begin{aligned} U_{i,1} &= 0 && i < 5 \\ &= (i - 5) \times 0.2 && i = 5 \text{ to } 10 \\ &= 1.0 && i > 10. \end{aligned}$$

A deterministic catch $C_{i,1}$ was obtained given F_1 and the population vector N_2 found. The catch and population vector in year 2 was then calculated with $F_2 = F_1$ and a changed $U_{i,2}$,

$$\begin{aligned} U_{i,2} &= 0 && i < k_2 - 5 \\ U_{i,2} &= (i + 5 - k_2) \times 0.2 && i = k_2 - 5 \text{ to } k_2 \\ U_{i,2} &= 1.0 && i > k_2. \end{aligned}$$

With $U_{i,3} = U_{i,1}$ and $F_3 = F_1$ a third catch was obtained.

From this simulation two recruitment values can be obtained, r_2 and r_3, using Equation (4). The results are given in Table 3 and demonstrate 1) the effect of recruitment occurring earlier in the second year (r_2 with $k_2 < 10$, and r_3 with $k_2 > 10$); 2) the effect of recruitment occurring later in the second year (the converse); and 3) the effect of the age of recruitment fluctuating about an average k to $\pm|k - k_2|$.

Under these conditions Equation (4) accurately gives the proportion of new recruits in the population and, as expected, if selection and recruitment occur at an earlier age in the second year then a large burst of new recruits will appear. If selection occurs much later, then even the recruits of the previous year will not be seen, giving the negative values. Such a feature was noted by Holt and de la Mare (1983). Horwood et al. (1985) fitted a selection pattern with age that was constant over time for minke whales of the Southern Hemisphere and presented the residual differences. A substantial switching of effort on to different age classes was found over a period of years, and it was shown that this was reflected in the calculated recruitment rates. These residuals and recruitment rates are shown in Table 4 and clearly illustrate the character of the estimate.

The problem is then not of calculation but of interpretation, in that we do not know selection has changed, and in using this technique it is assumed that the recruitment pattern is constant. A decreasing trend in recruitment rate will be interpreted as a decline in the true rate and not as an increasing age at recruitment and vice versa. As Table 3 shows these rates differ greatly from the 0.095 for constant selection, being much higher or lower depending on the trend in recruitment pattern. Consequently a systematic change in recruitment to the fishery will cause substantial problems in interpretation of the recruitment rates.

TABLE 3.—Recruitment rates calculated from Equation (4) for the model described in section vi. k is the age of first full recruitment used in Equation (4), k_2 is the first full recruitment in year 2. $\bar{r}$ is the average of the two values and nr is the average approximate net recruitment rate.

k_2	k	r_2	r_3	$\bar{r}$	nr
6	10	0.367	−0.301	0.033	−0.015
7	10	0.311	−0.194	0.058	0.009
8	10	0.247	−0.093	0.077	0.028
9	10	0.176	0.004	0.090	0.041
10	10	0.095	0.095	0.095	0.046
11	11	0.004	0.181	0.092	0.043
12	12	−0.099	0.260	0.080	0.031
13	13	−0.213	0.332	0.059	0.010
14	14	−0.340	0.397	0.028	−0.020

Table 3 also indicates what is likely to occur if the age of recruitment systematically fluctuates about a set pattern. It might be hoped that the r values would average to a useful measure of mean recruitment rate. For $k_2 = 6$ the recruitment occurs much earlier in year 2, giving a high r_2, and returns to normal in year 3, giving a low r_3. However, the average (0.033) is much smaller than the 0.095 and the approximate net recruitment rate is negative. A similar feature is seen for $k_2 = 14$, but as $|k_2 - 10|$ tends to zero the discrepancy is less. If the system fluctuated so that we had a series $k(t) = 10, 6, 10, 14$, and 10, an approximate average value of r would be the average of the four values of r on Table 3 (0.367, −0.301, −0.340, 0.397 = 0.031), and the approximate symmetry gives a similar feature of low average recruitment rates.

One way of using the recruitment rates would be to multiply the net recruitment rate by an estimated population size obtained over the same period to give a catch quota which should approximately stabilize the population. From the simulation the average of the recruited population in years 1, 3, and 4 has been found. This is very near to the average of the 4 years if the basic recruitment pattern is assumed for the second year. A catch was then found which would make the recruited population in year 5 the same

TABLE 4.—Direction of residuals after fitting a time constant selection at age to minke whale data showing switching of fishing selection across ages with time. Recruitment rate (r) values reflect this switching. (After Horwood et al. 1985.)

Age	1974/75	1975/76	1976/77	1977/78	1978/79	1979/80	1980/81	1981/82
1	+	+	+	−	+	−	−	−
2	+	+	+	−	+	−	−	+
3	−	+	+	−	+	+	−	+
4	−	+	+	−	+	+	−	+
5	−	−	+	+	−	−	−	+
6	−	−	+	+	−	−	−	+
7	−	−	+	+	−	−	−	−
8	−	−	+	+	−	−	−	−
9	−	−	−	+	−	+	+	−
10	−	−	−	+	−	+	+	−
11	−	−	−	+	−	+	+	−
12	+	−	−	+	−	+	+	−
13	−	−	−	+	−	+	+	−
14	+	−	−	+	−	+	+	−
15	+	+	−	+	−	+	+	−
16	−	+	−	+	−	+	+	−
r values	0·12	0·18	0·01	0·19	−0·08	−0·00	0·27	

size as this average. The ratio of this catch to the average population is the net recruitment rate that we would wish to use; these values varied from 0.04 (k_2 = 6) to 0.05 (k_2 = 14). This confirmed that the distortion of the age structure and population size by the change in selection had very little effect and that a value of r of 0.095, a net rate of 0.046, would be needed to calculate a stabilizing catch. The Table 3 averages are much smaller and we must conclude that if there is no trend in recruitment but a fluctuation of more than 1 year then the average estimated rates will be largely but undeterminably negatively biased, even if k is not underestimated. A 50-yr simulation confirmed this to be true.

As can be gleaned from the above, selection plays an important role in determining r. However, this technique treats overlapping pairs of years as being independent and implies a selection pattern for a pair of years, say 1980 and 1981 and a different one for years 1981 and 1982; these assumptions may be inconsistent. The difference may be small or large but there is no criterion for acceptability. Some current techniques take arrays of catch-at-age data and obtain best fits to the overall pattern (Beddington and Cooke 1981; Pope and Shepherd 1982), and Pope and Shepherd reduced consideration to two parameters. What is clear is that selection and recruitment or fishing rates are confounded, and these latter techniques make the assumptions clearly and would be expected to replace analyses of pairs of years.

CONCLUSIONS

If the recruitment pattern to the exploited population is constant then the following conclusions may be stated.

1. The "T" of Allen's technique is shown to be necessarily unity and this gives rise to Equation (4) for the estimation of recruitment rate.
2. If the age of first full recruitment is selected correctly then calculated recruitment rates are unbiased for changing fishing efforts or for an increasing or decreasing population.
3. If the age of first full recruitment is overestimated then an unbiased recruitment rate is found. If it is underestimated then a negative bias ensues. Inspection of Equation (4) however would caution use of an assumed high value of k, such that α and β were near unity, and this is reflected in the higher variances given by the approximate variance formula.
4. Aging bias and the use of age-length keys may spread the partially recruited age groups into allocated higher ages. The age of first full recruitment should be high enough to encompass this spreading.
5. No bias was detected in recruitment rates from a series of stochastic simulations although Allen (1981) found a small negative bias with low catches. As found by Allen (1981) coefficients of variation of the recruitment rates are high.
6. Equation (8) provides an approximate formula for the variance of the recruitment values given a fixed effort in the pairs of years. To use this the recruitment pattern needs to be estimated from the data as described by Allen (1966).

If the recruitment pattern is not constant, serious biases follow:

7. If there is a trend to earlier recruitment over a period of years high recruitment values will be seen and vice versa. These are likely to be interpreted as true increases or decreases.
8. If the recruitment pattern fluctuates about a mean then the net or gross average recruitment rate will be negatively biased, the bias increasing with the amplitude of the fluctuations. It is likely that many of the very low rates found by the International Whaling Commission are due to this feature.
9. These last two points indicate that for the technique to be useful it is necessary to establish that the recruitment pattern has been constant. This is likely to prove difficult and consequently much of the value of this simple method is lost.
10. For groups of years of data, alternative techniques should be investigated.
11. Finally it appears that the Allen recruitment rate, as calculated in this study or through Allen's original equations with T = 1, should be used with great care. It is subject to large and undeterminable biases and large variances. Where possible other techniques should be used.

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