INTRAGRAVEL FLOW AND INTERCHANGE OF WATER IN A STREAMBED

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ABSTRACT

The chemical quality of intragravel water in streams—the environment of salmon eggs, embryos, and alevins—is influenced by the rate of interchange of stream water and intragravel water. Factors controlling the direction and magnitude of flow or interchange of this water were identified in this study. Equations describing motion of waterflow within the streambed under specified boundary conditions are developed, and tests of the mathematical model with an electrolytic bath analog model are described.

The direction of waterflow within a streambed and the interchange of water between the bed and the stream depend primarily on the permeability, depth, and longitudinal profile of the porous streambed. Water upwells where permeability or depth of gravel decreases in the direction of streamflow and where the longitudinal bed profile is concave. Water downwells where permeability or depth of gravel increases in the direction of streamflow or where the longitudinal bed profile is convex.

Embryos and alevins of Pacific salmon (genus *Oncorhynchus*) live in gravel beds of streams for as long as 9 months before the fry emerge. The young salmon may be subjected to a poor chemical environment because of the low waterflow in the spawning bed. In laboratory experiments, oxygen privation and reduced waterflow, alone or together, impaired growth and development of salmonid embryos and caused mortality (Silver, Warren, and Doudoroff, 1963; Shumway, Warren, and Doudoroff, 1964). In natural streams, mortality of salmon spawn was high when the supply of dissolved oxygen in intragravel water (water occupying interstices within the streambed) became low (Phillips and Campbell, 1962; McNeil, 1966). Dissolved waste metabolites are known to be harmful to fishes: high levels of free carbon dioxide reduce the blood's affinity for oxygen, and un-ionized ammonium hydroxide is highly toxic to alevins (McNeil, Wells, and Brickell, 1964). The concentration of waste metabolites in the water surrounding organisms generating the wastes varies inversely with the rate of flow of intragravel water. It is, therefore, necessary that the velocity of intragravel flow be adequate to assure low concentrations of waste metabolite.

Because the primary source of oxygen and the water flowing through the gravel in many salmon spawning beds is stream water, the processes that regulate interchange between intragravel water and stream water are important in the ecology of salmon spawning beds. The purpose of my study is to identify some of the variables that control this interchange. In an earlier paper (Vaux, 1962), I related the direction of interchange to streambed shape from a mathematical model and presented field data which demonstrated that the relation was correct.

This paper treats the theory of flow of intragravel water in detail and gives the results of my laboratory experiments in which intragravel flow and interchange were simulated with an analog model. The means by which interchange can be controlled to improve water quality within salmon spawning beds are discussed.

I started my study of the movement of water within streambeds in 1958 while employed at the Fisheries Research Institute of the College of Fisheries, University of Washington. The experimental work and some of the mathematical formulation were completed in 1960–61 as part of a thesis, which was submitted to the Department

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of Chemical Engineering, University of Minnesota (Vaux, 1961).

All symbols are defined the first time they appear and are listed with definitions and units in the "Notation" section near the end of this paper.

THEORY OF INTRAGRAVEL WATERFLOW

Charles Slichter (1899) first applied the mathematics of partial differential equations to the motion of groundwater (waterflow in saturated soil). His formulation of the equations describing groundwater flow provided a foundation for subsequent analyses of similar problems of intragravel waterflow.

In developing the theory of intragravel waterflow, I treat the case where the rate equation for flow in porous media is linear, i.e., conditions under which Darcy's law applies. Ergun (1952) showed that for a given particle size and bed porosity, the ratio of pressure gradient to water velocity in a porous bed is given by $(\Delta P/L)/v \propto (c_1+c_2N_{Re})$ where c_1 and c_2 are known constants, and N_{Re} is the Reynolds number based on particle diameter. The constants c_1 and c_2 respectively account for pressure loss from the effects of viscous and kinetic energy. The expression $(\Delta P/L)/v \propto c_1$, called Darcy's law, is considered applicable for Reynolds numbers less than unity (Rumer, 1965). Although flow may be laminar in situations where the Reynolds numbers are higher than unity, the effects of convective acceleration in such situations invalidate Darcy's law (Silberman, 1965).

Analysis of data on gravel size, permeability, and hydraulic gradient from the Carmen-Smith spawning channel,[1] and Jones Creek, Alaska, spawning channels, McNary, Priest Rapids, and Robertson Creek,[2] and my data from Sashin Creek, Alaska, indicates that intragravel flow is of the linear laminar type and, therefore, described by Darcy's law. I assume that these cases describe the usual conditions in salmon spawning beds and that Darcy's law is generally applicable in the description of intragravel flow.

For situations in which the Reynolds numbers are less than unity, the velocity of laminar seepage flow within a porous medium, v, is related to the pressure gradient, $\Delta P/L$ by

$$v=-k\frac{g_c}{\mu}\frac{\Delta P}{L} \tag{1}$$

This relation is termed Darcy's law, but rather than a law, it is actually an equation which defines k, the "specific permeability," or just "permeability." In equation (1), μ is the liquid viscosity, and g_c is the constant of Newton's second law, introduced to make the equation dimensionally correct (g_c equals 1 g.-cm./dyne-sec.^{2} in centimeter-gram-second units).

The value of ΔP can be calculated by the relation $\Delta P=\rho\frac{g}{g_c}\ \Delta h$, where ρ is liquid density, Δh is head loss over the distance L, and g is acceleration due to gravity. Figure 1 illustrates the variables v, k, Δh, and L. The head loss, Δh, is shown in figure 1 by the elevations to which water rises in the piezometer tubes at opposite ends of the column of porous material.

We may extend Darcy's law to account for flow due to gravity, in addition to pressure, and write expressions for velocity in each of the three directions of a cartesian coordinate system. Ratios for $\Delta P/L$ can be replaced with the corresponding derivatives:

$$v_x=-k\frac{g_c}{\mu}\frac{\partial P}{\partial x} \tag{2a}$$

$$v_y=-k\frac{g_c}{\mu}\left(\frac{\partial P}{\partial y}+\frac{g\rho}{g_c}\right) \tag{2b}$$

$$v_z=-k\frac{g_c}{\mu}\frac{\partial P}{\partial z} \tag{2c}$$

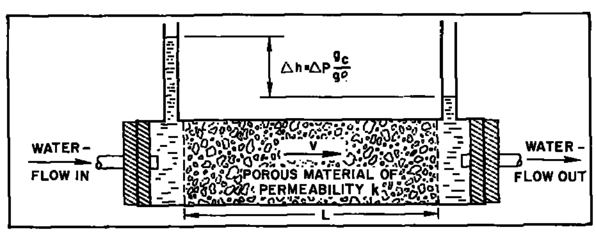

FIGURE 1.—Rectilinear waterflow through a sample of porous material showing a head loss $\Delta h=\Delta P\frac{g_c}{g\rho}$ over a distance L.

[1] Hagey, Dale W., and Robert T. Gunsolus, Progress report on the operation and evaluation of the Carmen-Smith spawning channel, 1960-64. Oregon Fish Commission, Feb. 1965, 12 pp.

[2] Data supplied by the Columbia Fisheries Program Office, Bureau of Commercial Fisheries, Portland, Oreg.

The coordinate system has been oriented so that the y direction is vertical and parallel to the direction of gravity. In equations (2) we assume that permeability is the same in every direction.

Equations (2) imply the single vector equation

$$\mathbf{i}v_x+\mathbf{j}v_y+\mathbf{k}v_z=-k\frac{g_c}{\mu}\left[\mathbf{i}\frac{\partial}{\partial x}(P)+\mathbf{j}\frac{\partial}{\partial y}\left(P+\frac{\rho g y}{g_c}\right)+\mathbf{k}\frac{\partial}{\partial z}(P)\right]$$

or

$$v=-k\frac{g_c}{\mu}\nabla\left[P+\frac{\rho g y}{g_c}\right] \tag{3}$$

where ∇ is the gradient operator defined by $\nabla\equiv\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$ and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors respectively in the three mutually perpendicular directions of the distance variables x, y, and z.

For later application of Darcy's law it is convenient to define the expression $\phi\equiv\frac{g_c P}{g\rho}+y$ and rewrite equation (3) as

$$v=-k\frac{\rho g}{\mu}\nabla\phi \tag{4}$$

Note that ϕ, the potential for intragravel flow, represents the total energy of intragravel water and is the sum of pressure energy and elevation energy. Kinetic energy is assumed to be negligible.

The continuity equation or equation of conservation of mass for an incompressible fluid is

$$\nabla\cdot\mathbf{v}=0 \tag{5}$$

The substitution of Darcy's law, equation (4), into equation (5) gives

$$k\nabla^2\phi+(\nabla k)\cdot\nabla\phi=0 \tag{6}$$

If the permeability, k, is homogeneous or uniform (i.e., independent of position) equation (6) reduces to Laplace's equation,

$$\nabla^2\phi=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{7}$$

Meaningful solutions of equation (6) or (7) cannot be found until the boundary conditions (i.e., the values of ϕ or derivatives of ϕ along the boundaries of the porous bed) are specified.

I have thus far discussed any laminar flow described by Darcy's law. Components of waterflow within a streambed are shown in figure 2. The porous streambed, A, is considered to be a stable bed consisting of particles and having a uniform permeability, k. The ambient stream, B, is continuous and uniform. The porous streambed is bounded below by an impermeable stratum, C. Any flow, D, within the porous bed is termed intragravel flow and is characterized by a velocity vector, $\mathbf{v}$. Any flow across the upper boundary of the bed is termed interchange. An upward interchange from the bed to the stream is upwelling, F; a downward interchange is downwelling, E.

The above conditions of uniform permeability and an impermeable stratum, for which equation (7) applies, characterize most manmade spawning channels. The permeability of natural streambeds usually decreases with depth, and flow is described by equation (6). Where permeability varies with depth, one may measure permeability at several depths and assign an appropriate permeability function, $k(y)$.

Equation (6) or equation (7) for uniform permeability characterizes all laminar flow through porous materials; the boundary conditions distinguish particular seepage situations. Our consideration of intragravel flow is limited to a two-dimensional system for which four boundary conditions must be specified about a bed of finite dimensions. An infinitely long bed requires specification of two boundary conditions. Because there is no flow across the lower boundary of the porous bed, the component of waterflow normal to the boundary at the boundary is 0 and

$$\left.\frac{\partial\phi}{\partial n}\right|_{\text{boundary},\ y_2(x)}=0 \tag{8a}$$

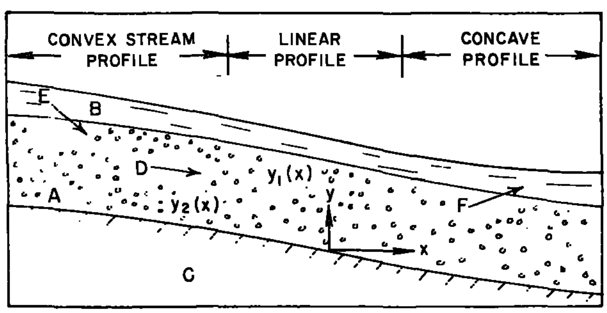

FIGURE 2.—Description of terms and properties relating to waterflow within a streambed. A, porous bed; B, ambient stream; C, impermeable stratum; D, intragravel flow; E, downwelling; F, upwelling; y_1, bed surface profile; y_2, impermeable stratum profile.

where n is the direction normal to the impermeable surface.

The velocity vector at any point within a streambed is influenced by streambed geometry and boundary conditions over the length of the stream. For investigation of intragravel flow in a finite reach of stream, between $x=0$ and $x=a$, we ignore upstream and downstream conditions. The boundary conditions $\phi(0,y)$ and $\phi(a,y)$ depend upon conditions over all the streambed. Since $\phi(x, y_1(x))$, hereafter denoted by $\phi(x)$, is unknown outside of $0 \leq x \leq a$, it is necessary to assign reasonable estimates of ϕ along the upstream and downstream boundaries of the isolated region. Accordingly, in isolating a region of streambed of depth b between $x=0$ and $x=a$, I specify the boundary conditions $\phi(0,y)$ and $\phi(a,y)$ to be constant and respectively equal to $\phi(0,b)$ and $\phi(a,b)$. The physical model is shown in figure 3(i)

$$\phi(0,y)=\phi(0,b) \qquad (8b)$$

$$\phi(a,y)=\phi(a,b) \qquad (8c)$$

Because of continuity of pressure across the stream-gravel interface, the potential at the upper bed boundary, $\phi[x, y_1(x)] \equiv \phi(x)$, must be equal to the potential at the bottom of the contiguous flowing stream. The problem of determining the potential along the upper boundary, $\phi(x)$, then becomes one of hydraulics.

The bottom pressure, $P(x,y_1)$, of a flowing stream is related to properties of the stream by

$$P(x,y_1)=\frac{\rho g}{g_c}d(x)+P_o+P_c(x,y_1) \qquad (9)$$

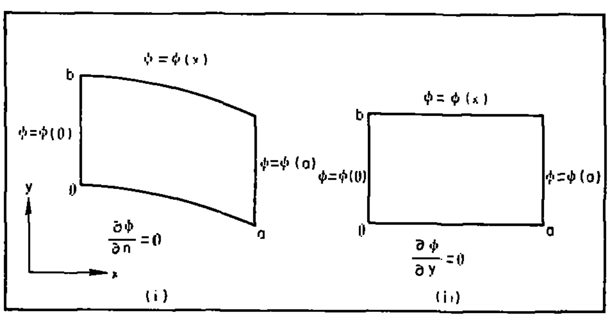

FIGURE 3.—(i) Boundary value problem representing the physical model for flow through a streambed of porous material. The shape of the region is the actual shape of the porous bed in which the upper and lower boundary curves are the topographical profiles of the bed surface and impermeable layer. (ii) Part (i) mapped into a rectangle.

in which $d(x)$ is the stream depth, P_o is atmospheric pressure, and P_c is centrifugal pressure due to stream curvature. From a stream-continuity equation, $d(x)U(x)=$constant, and the Manning flow formula,[3] $U(x)d(x)^{-2/3}\ y'1(x)^{-1/2} \simeq$constant, where y' is the hydraulic gradient and U is the average stream velocity, I find that the stream depth varies with the 0.3 power of hydraulic gradient. Considering small changes in stream gradient so that changes in $P_c(x)$ and $d(x)$ are small, it follows that changes in the right-hand side of equation (9) are small compared with changes in $y'(x)$ and that $g_cP/g\rho \simeq$constant, from which we may approximate the potential of the upper bed boundary

$$\phi(x)=y_1(x)+\frac{g_cP(x,y_1)}{g\rho} \simeq y_1(x)+\text{constant} \qquad (8d)$$

The applicability of equation (8d) is limited to reaches of nearly constant stream depth (see footnote 3). Because the average velocity of streamflow is several orders of magnitude greater than the velocity of intragravel flow, the influence of interchange on stream discharge is negligible.

Equation (6) or equation (7) for uniform permeability and the equations for the four boundary conditions (8a–8d) complete the boundary value problem. Figure 3(i) depicts the model that is considered for particular solutions. Solution of the boundary value problem is facilitated by assuming the bed to be of constant depth, b (so that $y_1=y_2+b$ (fig. 2)) and by mapping the irregularly bounded region into a rectangle, as in figure 3(ii). The error introduced is negligible because, over short distances (e.g., less than 10 m.), natural streambeds commonly have almost constant depth and are nearly horizontal. Boundary potentials are unchanged by formation of a rectangle.

Analytical solutions of the boundary value problem describing intragravel flow usually involve the use of infinite series, i.e., are of open form. Two simple closed-form solutions arise in the methods used here to solve the problem.

1. A potential $\phi(x)=\phi(0)-\phi(0)x/a+p\sin(\pi x/a)$ in which p is an arbitrary constant holds in a curved streambed surface profile for $0<x<a$. According to equation (8d) the profile has a

[3] This form of the Manning equation implies that the stream hydraulic radius is equal to $d(x)$, that bed roughness is constant in the direction of flow, and that the stream energy gradient is parallel to the streambed surface.

negative slope and is convex if $0<p\leq\phi(0)/\pi$, and concave if $0>p\geq-\phi(0)/\pi$ (See figure 2). The solution to the boundary value problem equation (8a-d) for this surface potential and a bed of uniform permeability is

$$\phi(x,y)=p\,\frac{\cosh(\pi y/a)}{\cosh(\pi b/a)}\sin(\pi x/a)+\phi(0)-\frac{\phi(0)x}{a} \tag{10}$$

If we define interchange as the vertical (y-directed) component of intragravel flow at the streambed surface (fig. 2) the exact magnitude of interchange is given by

$$\mathbf{v}_{y|y=b}=-k\,\frac{\rho g}{\mu}\frac{\partial\phi}{\partial y}\mathbf{j}_{\,|y=b} \tag{11}$$

For the example at hand the interchange velocity is

$$\mathbf{v}_{y|y=b}=-k\,\frac{\rho g}{\mu}\frac{\pi p}{a}\tanh\frac{\pi b}{a}\sin\frac{\pi x}{a}\mathbf{j} \tag{12}$$

This example shows that the direction of interchange depends on the shape of the streambed surface. A convex streambed profile for which p is positive induces a negative interchange velocity or downwelling; a concave profile for which p is negative causes upwelling.

2. Statement of the boundary value problem for an infinitely long streambed requires only equations (6), (8a), and (8d). We may solve the boundary value problem for the flow within an infinitely long streambed by the mathematics of complex variables (Weinberger, 1965, pp. 201–268). The transformation $\zeta=(\cosh Z-1)/\sinh Z$ conformally maps an infinitely long streambed of depth $b=\pi/2$ in the Z plane into a half disk in the ζ plane. The boundary potential $\phi(x)=-q\tanh x$ for $-\infty<x<\infty$ where q is a positive dimensionless, adjustable parameter is associated with the sigmoid-shaped streambed in figure 2. The solution of the boundary value problem is $\phi(x,y)=-q\sinh x/(\cosh x+\cos y)$. From equation (11) we find the interchange velocity to be

$$\mathbf{v}_{y|y=b}=kq\,\frac{\rho g}{\mu}\frac{\sinh x}{\cosh^2 x}\mathbf{j} \tag{13}$$

As in the previous example, downwelling takes place in a convex streambed profile $(-\infty<x<0)$ and upwelling in a concave profile $(0<x<\infty)$.

ANALOG MODEL INVESTIGATION OF INTRAGRAVEL FLOW

An electrolytic-bath analog model was used to solve the boundary value problem (equations 7 and 8a–8d) for several specified boundary conditions and a streambed of uniform permeability. This technique of solving the partial differential equations of intragravel flow rests upon the analogy between Darcy's and Ohm's laws and the respective steady-state conservation equations of mass and electrical charge. An electrolytic equivalent of a streambed section with intragravel flow can be prepared by constructing a shallow tray geometrically similar to the two-dimensional porous bed. The depth of the conducting liquid at any point in the tray is proportional to the permeability at the corresponding point in the porous bed. The conducting tray walls, usually metal strips, are set at voltages proportional to the analogous hydrodynamic flow potentials or values of ϕ (Zangar, 1953, p. 24).

EQUIPMENT AND PROCEDURE

The electrolytic-bath model constructed for this investigation was a 10- by 20-cm. plexiglass tray filled to a depth of 2.5 cm. with a 0.01 normal solution of aqueous sodium chloride (fig. 4). The end walls representing constant potential (constant ϕ) surfaces were brass strips; a nonconducting strip represented a water-impermeable surface. Rheostat-controlled voltages applied separately to each of 15 electrodes along the variable-potential wall ($y=b$) provided an approximation to the potential function, $\phi(x)$.

In all operations of the analog model, 10 volts of 60-cycle alternating current were impressed

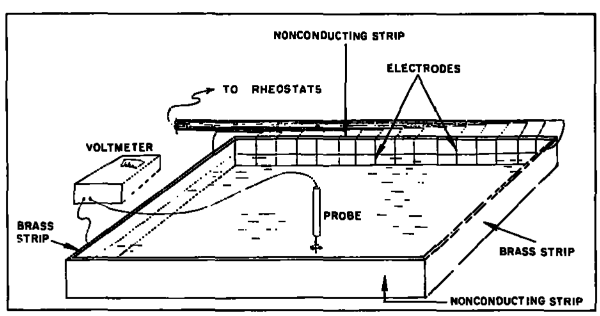

FIGURE 4.—Electrical circuits and physical layout for electrolytic-bath analog model used to investigate flow of intragravel water in a streambed.

between the brass strips; appropriate intermediate voltages were applied to the electrodes.

Solutions of the partial differential equation for different boundary conditions were obtained by tracing with a probe connected in series with a voltmeter and one tray wall along constant potential lines in the electrolyte. Results were recorded with a pantograph.

DIRECTION OF FLOW

Figures 5 through 9 represent longitudinal cross sections of a streambed: the top surface of the figure represents the upper surface of the streambed and the bottom, the impermeable stratum. The right and left ends of each figure represent arbitrary vertical planes bounding the reach of streambed considered. The lines of constant

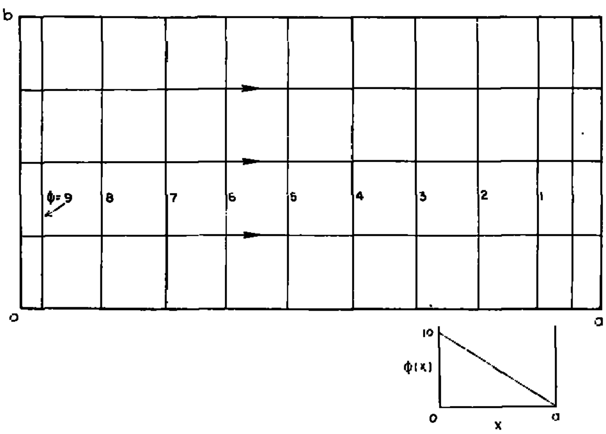

FIGURE 5.—Flow net for straight-line-surface potential $b/a=\frac{1}{2}$.

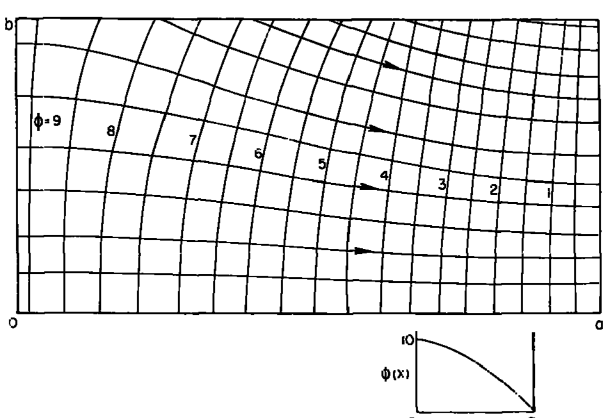

FIGURE 6.—Flow net for convex parabolic surface potential.

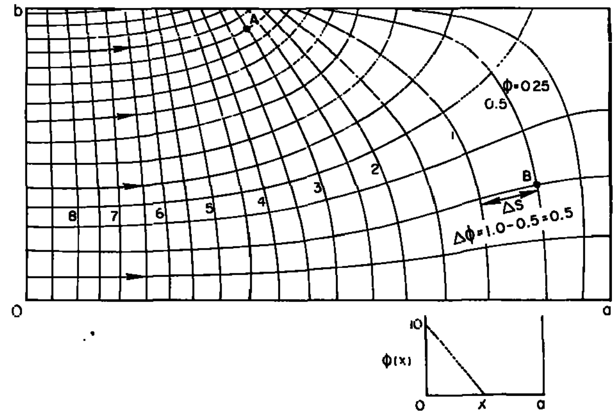

FIGURE 7.—Flow net for concave broken-line-surface potential, illustrating evaluation of Δs and $\Delta \phi$.

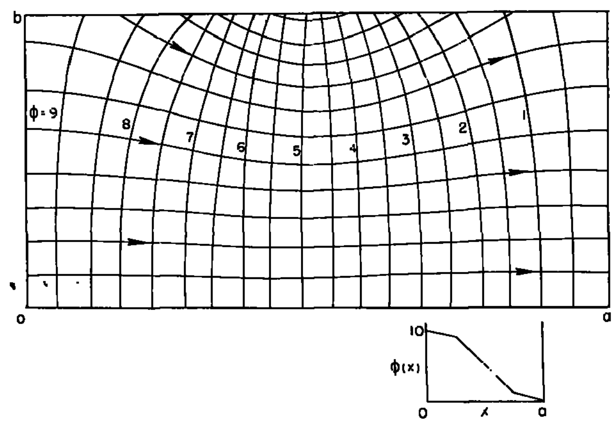

FIGURE 8.—Flow net for sigmoid surface potential.

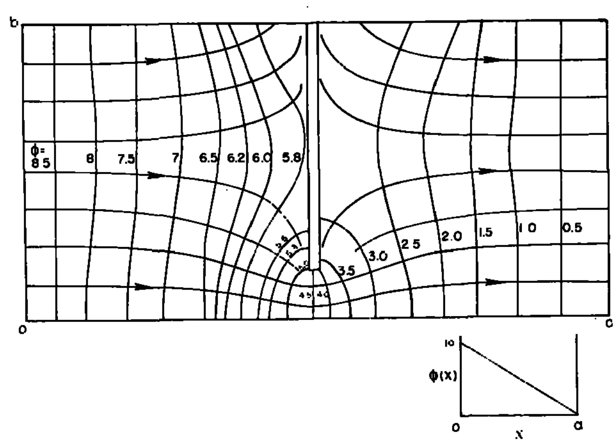

FIGURE 9.—Flow net for straight-line surface potential with impermeable intragravel barrier at $x=0.5a$, $b/a=\frac{1}{2}$.

potential, shown in the figures to be approximately vertical, were obtained from the analog model; the intragravel flow streamlines were later drawn normal to the constant potential lines to obtain a grid of "curvilinear squares" (Zangar, 1953, p. 17). Actually, the lines of constant potential shown are the analogs of the recorded lines of constant voltage, and the intragravel flow streamlines are the analogs of electrical current vectors. Figures 5, 6, 7, and 8 show the flow nets corresponding to certain of the boundary potential functions, $\phi(x)$, investigated. Figure 9 shows the effect of changed permeability on flow.

EFFECTS OF STREAMBED CHARACTERISTICS ON WATER INTERCHANGE

Accompanying each of the analog solutions shown in figures 5 through 9 is a small figure showing the function, $\phi(x)$, investigated. Since the flow potential along the open surface is approximately the elevation of the porous bed surface plus a constant, i.e., $\phi(x)=y_1(x)+$constant, each of the graphs is, in effect, a longitudinal profile of the streambed associated with the function. Figure 5 shows the pattern of intragravel flow within a streambed whose surface is straight. The profile of figure 6 may be viewed as a longitudinal convex stream section. Figure 7 illustrates the intragravel flow expected in the streambed beneath and beyond a rapids.

Figure 8 illustrates intragravel flow near a riffle between two low-gradient stream sections. This pattern of intragravel flow typifies the mechanism by which respiring salmon eggs and alevins are supplied with dissolved oxygen. Freshly oxygenated water downwells into the streambed, passes through the streambed's interior where oxygen is removed by salmon embryos or alevins, and upwells back into the stream.

The effect of placing a sheet piling in the gravel bed is shown by comparing the flow net of figure 5 with that of figure 9. Note that the streambed-surface potential, $\phi(x)$, in figure 9 is the same as that in figure 5 where, in the absence of a sheet piling, there is no interchange. The sheet piling directs intragravel flow to the surface upstream and induces downwelling downstream of the piling. The extent of interchange induced depends on the depth to which the piling is driven and the nearness of its bottom edge to an impermeable stratum.

Figures 5, 6, 7, and 8 show that the flow nets are characterized by variable directions of interchange: upwelling, downwelling, or interchange in both directions. Furthermore, the x-directed component of intragravel flow is always positive. From an analysis of many similar analog solutions and from observations in natural streams, I have found that downwelling occurs in longitudinally convex stream sections, as illustrated in figure 6. Similarly, upwelling takes place in longitudinally concave stream sections.[4]

Natural stream profiles usually contain alternate convex and concave reaches which cause alternate regions of upwelling and downwelling of stream water into and from the streambed. The penetration depth of this interchange flow depends on streambed geometry, e.g., the amount of curvature of the bed surface and depth of the streambed. In artificial spawning channels of uniform depth of streambed and uniform permeability, the bed surface is often groomed to an almost flat surface, and almost no potential exists for interchange. The movement of oxygen-rich water into the streambed must be by mechanisms other than normal water interchange, such as mechanical dispersion.

The flow nets of figures 5 through 9 show relative magnitude as well as direction of intragravel flow. Velocity of intragravel flow varies inversely with distance between lines of flow potential. When the lines are closely spaced, as at point A in figure 7, intragravel flow velocity is high. Widely spaced lines, as at point B in figure 7, show a low velocity of intragravel flow. Numerical values of velocity are calculated from equation (4), through use of a value of

$$|\nabla\phi|\simeq\frac{\Delta\phi}{\Delta s},$$

taken from the flow net as illustrated in figure 7 ($\Delta\phi$ is read as "the change in ϕ"). The symbol s is the distance variable along a streamline. The streamlines indicate the direction of intragravel flow.

[4] A. C. Cooper (1965), in a study of intragravel flow in salmon redds, showed results of intragravel backflow, $v_x<0$, and upwelling beneath a convex streambed surface. For this geometry the small radius of curvature of the streambed surface does not allow the approximation of equation (8d), and the boundary condition at the streambed surface, $\phi(x)$, must be obtained by a more rigorous approach. The geometry of Cooper's model studies suggests counter pressure gradients at the streambed surface and reaches of strongly variable streambed depth in the direction of flow. Such conditions would account for an upstream flow of intragravel water.

Errors in the technique included resistance heating of the electrolyte and polarization of the electrodes. The effect of polarization is shown where the potential gradient near the equipotential wall at $x=0$ is particularly steep.

The solutions of figures 5 through 9 for $b/a=1/2$ are not affine solutions; that is, they cannot be "stretched out" to describe exactly the intragravel flow in porous beds of different depth-length ratios. The flow nets shown, however, bear resemblance to and have the same general dependence on boundary-potential shape as rectangular beds of any depth-length ratio.

We have seen that for uniform permeability and constant bed depth, direction and magnitude of interchange depend upon configuration of the streambed surface. Direction and magnitude of interchange vary as well with longitudinal variations in depth and permeability of the streambed.

Consider the streambed section of figure 10. For this section the intragravel-flow streamlines will have a form somewhat like that shown in section C. Assume that all streamlines are parallel to the lower bed boundary, as in B of figure 10, and that water leaves or enters the streambed by a y-directed velocity (interchange) concentrated at the upper bed boundary, $y=b$ (see Lubyako, 1956, for discussion of this assumption). By a mass balance about the lamina of depth, b, width, w, and thickness, Δx, shown in A of figure 10,

$$\rho bwv_x|_x - \rho bwv_x|_{x+\Delta x} = v_y w \Delta x \rho. \tag{14}$$

In the limit as Δx approaches zero, the interchange, $v_y|_{y=b}$, is given by

$$v_y\Big|_{y=b} = -\frac{d}{dx}(bv_x). \tag{15}$$

Combining equation (15) with Darcy's law for the x-directed component of velocity

$$v_x = -k\frac{\rho g}{\mu}\frac{d\phi}{dx}, \tag{16}$$

and equation (8d) yields

$$v_y\Big|_{y=b} = \frac{\rho g}{\mu}\frac{d}{dx}\left(bk\frac{dy^1}{dx}\right). \tag{17}$$

Equation (17) agrees with the analytical and analog results for interchange direction. For constant bed depth, d, and permeability, k,

$$v_y\Big|_{y=b} = \frac{\rho g}{\mu}bk\frac{d^2y_1}{dx^2}. \tag{18}$$

For a longitudinally convex streambed surface, d^2y_1/dx^2 is negative, and equation (18) implies a downwelling. Conversely, equation (18) indicates upwelling with a concave streambed surface.

From equation (17) I have summarized in table 1 the dependence of interchange upon streambed permeability and geometry, both of which may be regulated to control the direction of interchange. Assuming that the permeability of a streambed may be increased by removing fine materials (Krumbein and Monk, 1942; McNeil and Ahnell, 1964) or that the effective gravel-bed depth (i.e., depth to which intragravel flow can occur) may be increased by removing fine particles from lower strata, it should be possible to create the interchange patterns shown in figures 11, 12, and 13. It should also be possible to control direction and amount of interchange in salmon spawning areas by varying the streambed-surface profile (fig. 14).

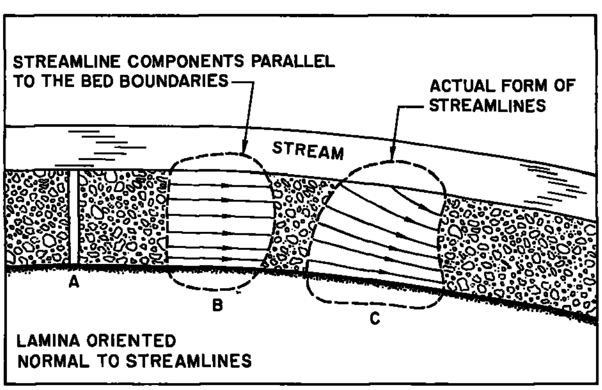

FIGURE 10.—Idealized concept of intragravel flow in a streambed.

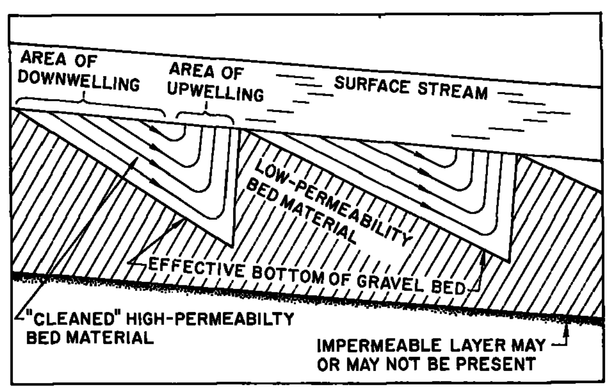

FIGURE 11.—Interchange induced by variation of depth of the streambed.

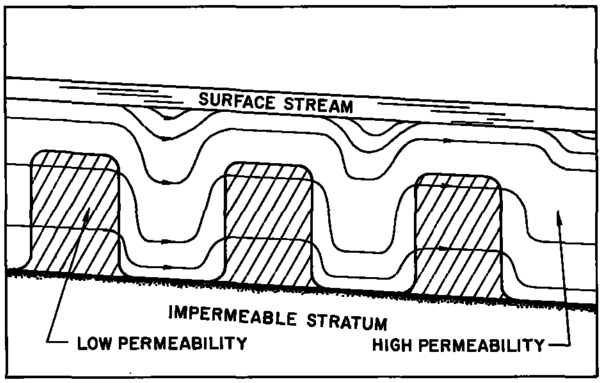

FIGURE 12.—Interchange induced by creating adjacent regions of high and low permeability.

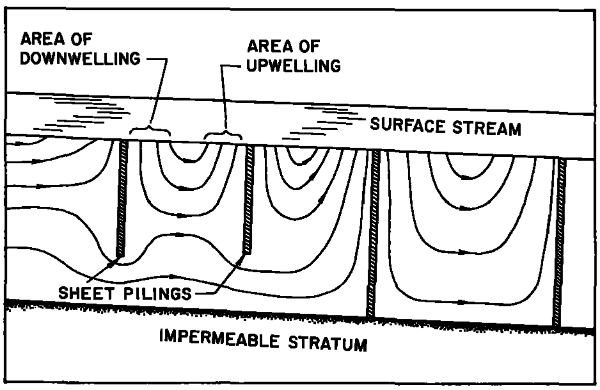

FIGURE 13.—Interchange induced by sheet pilings.

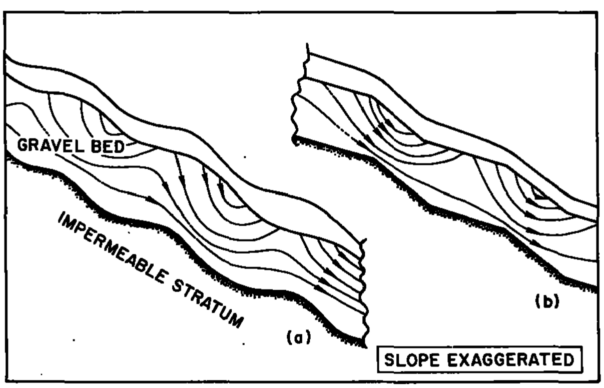

FIGURE 14.—Interchange established in streambeds by shaping the gravel bed into (a) a rounded profile and (b) an angular profile.

CONCLUSIONS

A constantly changing supply of well-oxygenated water is required in salmon spawning beds for proper survival, growth, and development of salmon embryos and alevins. Interchange between stream and intragravel water is essential for the delivery of adequate dissolved oxygen to pre-emergent fish. Knowledge of the factors that control interchange aids in understanding mortality factors and in developing means of increasing production of salmon fry from spawning beds.

TABLE 1.—*Summary of effects of permeability, depth, and surface profile of streambed on interchange*

Streambed characteristics	Causes upwelling	Causes downwelling	Illustrated in figure(s)
Bed depth, $b(x)$	Decreasing in the direction of flow.	Increasing in the direction of flow.	11.
Permeability, $k(x)$	do	do	12, 13.
Bed-surface profile, $y_1(x)$ (see fig. 2).	Concave	Convex	6, 7, 8.

Waterflow within a spawning bed can be described by Darcy's law and an equation of conservation of mass for an incompressible fluid. The potential causing intragravel flow,

$$\phi = y + g_c P/\rho g,$$

is the solution of the equation resulting from combination of Darcy's law and the mass conservation equation.

The potential along the stream-streambed boundary is described approximately by

$$\phi(x) = y_1(x) + \text{constant},$$

where $y_1(x)$ describes the profile of the streambed surface.

Interchange is controlled by the shape of the longitudinal profile of the streambed and longitudinal gradients of permeability and depth of the gravel bed according to the approximation

$$v_y \simeq \frac{Pg}{\mu} \frac{d}{dx}\left(kb\frac{dy_1}{dx}\right).$$

A concave streambed surface induces upwelling, and a convex surface induces downwelling. Increasing permeability of the streambed (measured in the direction of waterflow) induces downwelling, and decreasing permeability induces upwelling. Increasing bed depth (measured in

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the direction of waterflow) causes downwelling, and decreasing bed depth causes upwelling. The relation between direction of interchange and the shape of the streambed surface profile of the spawning bed was tested with an electrolytic-bath analog model and found to be correct.

The results of this work clearly demonstrate that the direction and magnitude of interchange in salmon spawning beds can be controlled by simple alteration of one or more of the three characteristics: (1) the surface profile of the bed, (2) the bed depth, and (3) the bed permeability. Selective removal of fine particles from "patches" of spawning gravel or the driving of impermeable sheet pilings could, for example, increase interchange and the amount of dissolved oxygen delivered to eggs and alevins.

NOTATION

Symbol	Notation	Units (Length, force, mass, time)
a	Length of reach of streambed.	L
b	Depth of streambed.	L
c_1, c_2	Constants of Ergun's equation.	FtL^{-4}
d	Stream depth.	L
g	Acceleration due to gravity.	Lt^{-2}
g_c	Newton's law constant, defined by force$=\frac{1}{g_c}$ mass$\times$acceleration.	$MLF^{-1}t^{-2}$
Δh	Head loss.	L
$\mathbf{i}$	Unit vector in x direction.	
$\mathbf{j}$	Unit vector in y direction.	
$\mathbf{k}$	Unit vector in z direction.	
k	Permeability, a function of (x, y, z).	L^2
L	Length of permeameter.	L
n	Distance variable in direction normal to a surface.	L
N_{Re}	Reynold's number based on particle diameter.	
p	Parameter for adjusting boundary potential in equation (10).	Dimensionless
P	Pressure.	FL^{-2}
q	A positive adjustable parameter, (equation (13)).	Dimensionless
s	Distance variable along a streamline.	L
$U(x)$	Average stream velocity.	Lt^{-1}
v	Seepage velocity.	Lt^{-1}
w	Width of streambed.	L
x, y, z	Distance variables along axes.	L
y_1	Elevation of streambed surface.	L
y_2	Elevation of surface of impermeable stratum.	L
Z	Complex variable$=x+\sqrt{-1}\,y$.	
ζ	Complex variable in image plane.	
μ	Liquid viscosity.	$ML^{-1}t^{-1}$
ρ	Water density.	ML^{-3}
ϕ	Potential for intragravel flow.	L
$\phi(x)$	Potential at bed surface $=\phi[x, y, (x)]$.	L
	$\nabla=\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}$, the gradient operator.	L^{-1}
	$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, the laplacian operator.	L^{-2}

NOTES

Vectors are denoted by boldface letters, e.g., $\mathbf{v}$ is a velocity vector

$\equiv$ is read as "is defined as"

$\simeq$ is read as "approximately equal to"

Δ is read as "the change in"

A vertical bar is read as "evaluated at," e.g.,$|_{y=b}$ = "evaluated at $y=b$"

$\propto$ is read as "is proportional to"

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