Supplementary materials

Additional aspects of the projection model

The catch and bycatch are predicted using the equations

\[ \hat{C}_{t,j} = \frac{F_t s^T_{t,j} s^T_{t,j}}{z_{t,j}} N_{t,j} e^{-y_t M} (1 - e^{-z_{t,j}}) \]  
\[ \bar{D}_{t,j} = 0.2 \frac{F_t s^T_{t,j} (1 - s^T_{t,j})}{z_{t,j}} N_{t,j} e^{-y_t M} (1 - e^{-z_{t,j}}) \]  
\[ \bar{T}_{t,j} = 0.65 \frac{F_t s^T_{j,T}}{z_{t,j}} N_{t,j} e^{-y_t M} (1 - e^{-z_{t,j}}) \]  

where \( z_{t,j} \) is total fishery-related mortality on animals in size-class \( j \) during year \( t \):

\[ z_{t,j} = F_t s^T_{t,j} s^T_{t,j} + 0.2 F_t s^T_{t,j} (1 - s^T_{t,j}) + 0.65 F_t s^T_{j,T} \]  

\( F_t \) is full selection fishing mortality in the directed pot fishery, \( F_t s^T_{j,T} \) is full selection fishing mortality in the trawl (bycatch) fishery during year \( t \), \( s^T_{t,j} \) is total (logistic) selectivity for animals in size-class \( j \) in the trawl fishery, \( s^T_{j,T} \) is selectivity for animals in size-class \( j \) in the directed pot fishery during year \( t \), \( y_t \) is the time from 1 July to the mid-point of the fishery during year \( t \). Pot bycatch mortality of 0.2 and groundfish bycatch mortality of 0.65 [average of trawl (0.8) and fish pot (0.5) mortality] are assumed (Siddeek et al., 2019).

The size-transition matrix \( X \) is modeled as follows:

\[ X_{i,j} = \begin{cases} 
0 & \text{if } j < i \\
P_{i,j} + (1 - m_i) & \text{if } j = i \\
P_{i,j} & \text{if } j > i 
\end{cases} \]  

where:

\[ P_{i,j} = \begin{cases} 
\int_{-\infty}^{L_i-j} N(x|\mu_i, \sigma^2) \, dx & \text{if } j = i \\
\int_{L_i-j}^{L_i} N(x|\mu_i, \sigma^2) \, dx & \text{if } j < i < n, \\
\int_{L_i-j}^{\infty} N(x|\mu_i, \sigma^2) \, dx & \text{if } i = n 
\end{cases} \]

\[ N(x|\mu_i, \sigma^2) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu_i)^2}{2\sigma^2}}, \]

where \( \mu_i \) is the mean growth increment for crab in size-class \( i \), \( \mu_i = \omega_1 + \omega_2 \bar{L}_i \), \( \omega_1, \omega_2 \), and \( \sigma \) are estimated parameters, \( j_i \) and \( j_2 \) are the lower and upper limits of size-class \( j \), \( \bar{L}_i \) is the midpoint of the size-class \( i \), and \( m_i \) is the molt probability for size-class \( j \) estimated when \( j > i \):

Siddeek et al.: Development of harvest control rules for hard-to-age crab stocks  Fish. Bull. 118
\[ m_i = \frac{1}{1 + e^{c(t-a)}} \]  

(A6)

where \( c \) and \( d \) are parameters.

Catch-per-unit effort (CPUE) is calculated using the equation:

\[ CPUE_{t}^c = q \sum_j S_j^T S_j^r (N_{t,j} - 0.5 \left[ \hat{C}_{t,j} + \hat{D}_{t,j} + \hat{T}_{t,j} \right] ) e^{-y_t M} \]  

(A7)

where \( q \) is the catchability coefficient.

The size-structured assessment model

Basic population dynamics

Annual [male] abundances by size are modeled using the equation:

\[ N_{t+1,j} = \sum_{i=1}^{I} [N_{t,i} e^{-M} - (\hat{C}_{t,i} + \hat{D}_{t,i} + \hat{T}_{t,i}) e^{(y_t-1)M}] X_{i,j} + R_{t+1,j} \]  

(B1)

where the symbols are defined in Appendix A.

Initial abundance

The initial conditions are computed under the assumption that the population was in equilibrium in 1960, i.e.:

\[ N_{1960} = (I - XS)^{-1} R \]  

(B2)

where \( X \) is the size-transition matrix; \( S \) is a matrix with diagonal elements given by \( e^{-M} \); \( I \) is the identity matrix; and \( R \) is the product of average recruitment during 1987-2012 and the relative proportion of total recruitment to each size-class.

Selectivity and retention

Selectivity and retention are both assumed to be logistic functions of length. Selectivity changes over time for directed pot fishery total selectivity, but not for the retention curve:

\[ S_i = \frac{1}{1 + e^{\left[-ln(19) \frac{\theta_95-\theta_50}{\theta_95-\theta_50}\right]}} \]  

(B3)

where \( \theta_{95} \) and \( \theta_{50} \) are the parameters of the selectivity/retention patterns.

Recruitment

Recruitment to size-class \( i \) during year \( t \) is modeled as:

\[ R_{t,i} = \bar{R} e^{\epsilon_t} \Omega_i \]  

(B4)

where \( \Omega_i \) is a normalized gamma function; \( \bar{R} \) is the estimated (median) recruitment; and \( \epsilon_t \) is the (estimated) recruitment deviation for year \( t \).  

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Parameter estimation and likelihood function

Suppl. Table B1 lists the parameters of the model, indicating which are estimated and which are pre-specified. The objective function includes contributions related to the fit of the model to the available data and penalties (priors on various parameters). Suppl. Table B2 lists the weights for the components of the objective function.

Catches

The contribution of the catch data (retained, total, and groundfish discarded) to the objective function is given by:

\[
LL_{\text{Catch}}^r = \lambda_r \sum_t \left\{ \ln(\sum_j \hat{C}_{t,j} w_j + c) - \ln(\sum_j C_{t,j} w_j + c) \right\}^2
\]

\[
LL_{\text{Catch}}^T = \lambda_T \sum_t \left\{ \ln(\sum_j \hat{T}_{t,j} w_j + c) - \ln(\sum_j T_{t,j} w_j + c) \right\}^2
\]

\[
LL_{\text{Catch}}^{GD} = \lambda_{GD} \sum_t \left\{ \ln(\sum_j \hat{T}_{t,j} w_j + c) - \ln(\sum_j T_{t,j} w_j + c) \right\}^2
\]

where \(\lambda_r\), \(\lambda_T\), and \(\lambda_{GD}\) are the weights assigned to likelihood components for the directed pot retained, directed pot total, and groundfish discard catches; \(C_{t,j}\), \(T_{t,j}\), and \(T_{t,j}\) are, respectively, the observed numbers of crab in size-class \(j\) for the directed pot retained, the directed pot total, and the groundfish discard fisheries during year \(t\); and \(c\) is a small constant set to 0.001. There is an additional retained catch likelihood component (using Equation Supp. A5a without \(w\)) for the retained catch in number of crabs during 1981/82 to 1984/85 (catches are in weight are not available before 1985/86 fishing year).

Catch-rate indices

The catch-rate indices are assumed to be lognormally distributed about the model predictions. Account is taken of variation in addition to that related to sampling variation:

\[
LL_{\text{CPUE}} = \lambda_{r,CPUE} \left\{ 0.5 \sum_t \ln[2\pi(\sigma_{r,t}^2 + \sigma_e^2)] + \sum_t \frac{\left(\ln(CPUE_t^r + c) - \ln(CPUE_{t,\hat{r}}^r + c)\right)^2}{2(\sigma_{r,t}^2 + \sigma_e^2)} \right\}
\]

where \(CPUE_t^r\) is the standardized retained catch-rate index for year \(t\) (Siddeek et al., 2019); \(\sigma_{r,t}\) is standard error of the logarithm of \(CPUE_t^r\); \(\sigma_{r,t}^2 = \ln(1 + CV_{r,t}^2)\) where \(CV_{r,t}\) is the CV of the standardized retained fishery CPUE index for year \(t\); and \(\hat{CPUE}_{t,\hat{r}}^r\) is the model-estimate of \(CPUE_t^r\), i.e.:

\[
CPUE_{t,\hat{r}}^r = q_k \sum_j S_j^T S_j^r \left( N_{t,j} - 0.5[C_{t,j} + T_{t,j} - T_{t,j}] \right) e^{-y_t M}
\]
where $q_k$ is the catchability coefficient during the $k$-th time period (e.g., pre- and post-crab rationalization), $\sigma_e$ is the extent of over-dispersion, $c$ is a small constant to prevent zero values (set to 0.001), and $\lambda_{r,CPUE}$ is the weight assigned to the catch-rate data.

**Size-composition**

The size-composition data are included in the likelihood function using the robust normal for proportions likelihood, i.e.:

$$LL^F_{t} = 0.5 \sum_t \sum_j \ln(2\pi \sigma_{t,j}^2) - \sum_t \sum_j \ln\left[\exp\left(-\frac{(p_{t,j} - \hat{p}_{t,j})^2}{2\sigma_{t,j}^2}\right) + 0.01\right]$$

(B8)

where $P_{t,j}$ is the observed proportion of crabs in size-class $j$ in the catch for year $t$; and $\hat{P}_{t,j}$ is the model-estimate corresponding to $P_{t,j}$. For retained, total, and groundfish discard catch compositions, the proportions are computed as follows:

$$\hat{L}_{t,j}^R = \frac{\hat{c}_{t,j}^R}{\sum_i \hat{c}_{t,j}^R}, \quad \hat{L}_{t,j}^T = \frac{\hat{t}_{t,j}^T}{\sum_i \hat{t}_{t,j}^T}, \quad \hat{L}_{t,j}^{GF} = \frac{\hat{T}_{t,j}^{GF}}{\sum_i \hat{T}_{t,j}^{GF}}$$

(B9)

$\sigma_{t,j}^2$ is the variance of $P_{t,j}$:

$$\sigma_{t,j}^2 = \left[(1 - P_{t,j})P_{t,j} + \frac{0.1}{n}\right] / S_t$$

(B10)

and $S_t$ is the effective sample size for year $t$ and $n$ is the number of size classes.

**Tagging data**

Let $V_{j,t,y}$ be the number of tagged male crab that were released during year $t$ that were in size-class $j$ when they were released and were recaptured after $y$ years, and $\rho_{j,t,y}$ be the vector of recaptures by size-class from the males that were released in year $t$ that were in size-class $j$ when they were released and were recaptured after $y$ years. The log-likelihood corresponding to the multinomial distribution for the tagging data is then:

$$lnL = \lambda_{y,tag} \sum_j \sum_t \sum_y \mu_{j,t,y} l \ln \hat{\rho}_{j,t,y,i}$$

(B11)

where $\lambda_{y,tag}$ is the weight assigned to the tagging data for recapture year $y$, $\hat{\rho}_{j,t,y,i}$ is the proportion in size-class $i$ of the recaptures of males that were released during year $t$ that were in size-class $j$ when they were released and were recaptured after $y$ years:

$$\hat{\rho}_{j,t,y} \propto S^T [X]^T Z^{(j)}$$

(B12)

where $Z^{(j)}$ is a vector with $V_{j,t,y}$ at element $j$ and 0 otherwise; and $S^T$ is the vector of total selectivity for tagged male crab by the directed pot fishery. This log-likelihood function is
predicated on the assumption that all recaptures are in the direct pot fishery and the reporting rate is independent of the size of crab.

**Penalties**
Penalties are imposed on the deviations of annual pot fishing mortality about mean pot fishing mortality, annual trawl fishing mortality about mean trawl fishing mortality, and recruitment about mean recruitment.

\[ P_1 = \lambda_F \sum_t (\ln F_t - \ln \bar{F})^2 \]  
\[ P_2 = \lambda_{FTr} \sum_t \left( \ln F_{tTr} - \ln \bar{F}_{tTr} \right)^2 \]  
\[ P_3 = \lambda_R \sum_t (\ln \epsilon_t)^2 \] 

(B13)  
(B14)  
(B15)
Supplementary Table B1. Pre-specified and estimated parameters of the population dynamics model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fishing mortalities:</strong></td>
<td></td>
</tr>
<tr>
<td>Pot fishery, $F_t$</td>
<td>1981–2018 (estimated)</td>
</tr>
<tr>
<td>Mean pot fishery fishing mortality, $\overline{F}$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td>Groundfish fishery, $F^{Tr}_t$</td>
<td>1989–2018 (mean $F$ for 1989 to 1994 was used to estimate groundfish discards back to 1981 (estimated)</td>
</tr>
<tr>
<td>Mean groundfish fishery fishing mortality, $\overline{F}^{Tr}$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td><strong>Selectivity and retention:</strong></td>
<td></td>
</tr>
<tr>
<td>Pot fishery total selectivity, $\theta_{50}^T$</td>
<td>2 (1981–2004; 2005+) (estimated)</td>
</tr>
<tr>
<td>Pot fishery total selectivity difference, $\delta \theta^T$</td>
<td>2 (1981–2004; 2005+) (estimated)</td>
</tr>
<tr>
<td>Pot fishery retention, $\theta_{50}^r$</td>
<td>1 (1981+) (estimated)</td>
</tr>
<tr>
<td>Pot fishery retention selectivity difference, $\delta \theta^r$</td>
<td>1 (1981+) (estimated)</td>
</tr>
<tr>
<td>Groundfish fishery selectivity</td>
<td>fixed at 1 for all size-classes</td>
</tr>
<tr>
<td><strong>Growth:</strong></td>
<td></td>
</tr>
<tr>
<td>Expected growth increment, $\omega_1, \omega_2$</td>
<td>2 (estimated)</td>
</tr>
<tr>
<td>Variability in growth increment, $\sigma$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td>Molt probability, $a$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td>Molt probability, $b$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td>Natural mortality, $M$</td>
<td>1 (pre-specified, 0.21yr$^{-1}$)</td>
</tr>
<tr>
<td><strong>Recruitment:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of recruiting length-classes</td>
<td>5 (pre-specified)</td>
</tr>
<tr>
<td>Mean recruit length, $\overline{l}_R$</td>
<td>1 (pre-specified, 110-mm CL)</td>
</tr>
<tr>
<td>Distribution to size-class, $\beta_r$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td>Median recruitment, $\overline{R}$</td>
<td>1 (estimated)</td>
</tr>
<tr>
<td>Recruitment deviations, $\epsilon_t$</td>
<td>59 (1961–2019) (estimated)</td>
</tr>
<tr>
<td>Additional CPUE indices standard deviation, $\sigma_e$</td>
<td>1 (estimated)</td>
</tr>
</tbody>
</table>
Supplementary Table B2. The likelihood weights.

<table>
<thead>
<tr>
<th>Data and Penalty</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Catch:</strong></td>
<td></td>
</tr>
<tr>
<td>Total catch for 1990–2018, $\lambda_T$</td>
<td>Number of observer sampled pots scaled to a maximum 250</td>
</tr>
<tr>
<td>Groundfish bycatch for 1989–2018, $\lambda_{GD}$</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Catch-rate:</strong></td>
<td></td>
</tr>
<tr>
<td>Observer catch-rate for 1995–2018, $\lambda_{r,CPUE}$</td>
<td>1</td>
</tr>
<tr>
<td>Fish ticket retained crab catch-rate for 1985–1998, $\lambda_{r,CPUE}$</td>
<td>1</td>
</tr>
<tr>
<td><strong>Penalty:</strong></td>
<td></td>
</tr>
<tr>
<td>Pot fishing mortality dev, $\lambda_F$</td>
<td>Initially 1000, relaxed to 0.001 at phases $\geq$ selectivity phase</td>
</tr>
<tr>
<td>Groundfish fishing mortality dev, $\lambda_{FTr}$</td>
<td>Initially 1000, relaxed to 0.001 at phases $\geq$ selectivity phase</td>
</tr>
<tr>
<td>Recruitment, $\lambda_R$</td>
<td>2</td>
</tr>
<tr>
<td>Tagging likelihood</td>
<td>tag data distribution</td>
</tr>
</tbody>
</table>