Supplementary text

The potentially complicated relationships of availability and efficiency with catchability and selectivity

Describing a simple set of relationships between the pair of terms *availability* and *efficiency*, as typically used by survey scientists, and the pair of terms *catchability* and *selectivity*, as typically used by assessment scientists, would help to guide research on the functionality of sampling gear and the use of survey results in stock assessment models. Unfortunately, these relationships have the potential to be complicated, except in very special circumstances.

Either availability or efficiency, or both, may vary with respect to fish age, fish length, or both. To keep the notation from becoming too complicated, it will be assumed here that both availability and efficiency vary with fish age, but the argument would be the same if they varied with length or with both age and length.

To keep things simple, assume that the survey in question is a bottom trawl survey and that stations are selected on the basis of a simple random sampling design within a defined survey area. For any given tow and any given age, there will be some true biomass of fish in the water column above the ground swept by the net. Availability is typically defined as the proportion of the *true* biomass that the net *encounters*. Efficiency is typically defined as the proportion of the *available* biomass that the net *actually captures*.

As typically used by assessment scientists, catchability and selectivity both involve the following ratio:

$$ratio_{age} = \frac{survey\ total\ biomass_{age}}{true\ total\ biomass_{age}},$$

where *total* is included as a modifier of *biomass* in both the numerator and denominator above to emphasize that these quantities pertain to the entire stock, not just to an individual tow.

Catchability is typically defined as the maximum (across age) value of the above ratio, and selectivity is typically defined in the following way:

$$selectivity_{age} = \frac{ratio_{age}}{catchability}.$$

Case 1

An extremely simple case can be defined by the following assumptions: 1) both availability and efficiency are constant values across tows, 2) the stock is distributed evenly throughout its range, and 3) the survey area is equal to the distribution range of the stock. In this case, the survey's estimate of biomass at any given age can be expressed by the following equation:

$$survey\ total\ biomass_{age} = availability_{age} \times efficiency_{age} \times true\ total\ biomass_{age}.$$

Given the very restrictive assumptions specified for this case, when the right-hand side of the above is substituted into the ratio of survey total biomass to true total biomass, the true total biomass terms cancel, and the ratio is simply the product of availability at age and efficiency at age.

Therefore, although neither availability nor efficiency is *equivalent* to either catchability or selectivity in this special case, both of the latter terms can be defined easily in terms of the *product* of the former. Even in this extremely simplified case, however, it is important to recognize that selectivity at age does not necessarily vary directly with either availability at age or efficiency at age because it is the *product* of availability and efficiency that matters.

Case 2

If the extreme assumptions imposed in case 1 are relaxed, the relationships become more complicated. For example, if it is no longer assumed that the stock is distributed evenly throughout its range and that the survey area is equal to the range of the stock, then the survey estimate of biomass at any given age can be expressed with the following equation:

$$survey\ total\ biomass_{age} = availability_{age} \times efficiency_{age} \times \left(\frac{total\ survey\ area}{ntows}\right) \times \sum_{tow=1}^{ntows} \frac{true\ tow\ biomass_{tow,age}}{area\ swept_{tow}}.$$

Therefore, the survey estimate of total biomass at age is no longer proportional only to the product of availability at age and efficiency at age because this product is multiplied by another term that varies with age. When the ratio between survey total biomass and true total biomass is computed, this additional age-dependent term remains in the numerator and therefore neither catchability nor selectivity can be defined simply in terms of availability and efficiency.

Case 3

It is also possible that both availability and efficiency vary not only by age but also by tow (perhaps reflecting location- or time-specific environmental conditions, pure random variability, or some combination thereof). In this case, the survey estimate of biomass at any given age can be expressed by this equation:

$$survey\ total\ biomass_{age} = \left(\frac{total\ survey\ area}{ntows}\right) \times \sum_{tow=1}^{ntows} \left(availability_{tow,age} \times efficiency_{tow,age} \times \left(\frac{true\ tow\ biomass_{tow,age}}{area\ swept_{tow}}\right) \right).$$

Here, the survey estimate of total biomass at age is no longer proportional to the product of availability and efficiency in any simple way whatsoever because the product is tow-dependent and is incorporated into the estimate as a weighted average, with the weights both tow- and age-dependent. When the ratio between survey total biomass and true total biomass is computed, these complications remain in the numerator.

Conclusion

Even in the simplest case, there is no one-to-one correspondence between either availability or efficiency and either catchability or selectivity. As simplifying assumptions are relaxed, the relationships can become very complicated. For example, it is quite possible for selectivity to decline with age even when the product of average (across tows) availability and average (across tows) efficiency increases with age. Therefore, care should be taken when attempting to infer the shape of the selectivity function from evidence pertaining to availability or efficiency.